Stat 134 Final Review (Sec 1, Fall 2006) E. Mossel.

Name and SID number: ______________________________

Solve 5 of the following 6 problems.

1. The horn of an auto operates on demand 99% of the time. Assume each time you hit the horn, it works or fails independently of all other times.
[**a** (10 points)]. How many times do you expect to be able to honk the horn with 75% probability of not having any failures.
[**a** (10 points)]. What is the expected number of times you hit the horn before the tenth failure?

2.
[**a** (10 points)]. Show that if T has the Weibull (λ, α) distribution with the following density:

$$f(t) = \lambda \alpha t^{\alpha-1} e^{-\lambda t^{\alpha}} \quad (t > 0),$$

where $\lambda > 0$ and $\alpha > 0$ then T^{α} has an exponential λ distribution.
[**b** (10 points)]. Show that if U is uniform $(0,1)$ random variable then $T = (-\lambda^{-1} \log U)^{1/\alpha}$ has a Weibull (λ, α) distribution.

3. Let Y be the minimum of 4 independent random variables with uniform distribution on $(0,1)$ and let Z be their maximum. Find:
[**a** (10 points)]. $P(Z \le 3/4 | Y \ge 1/4)$.
[**b** (10 points)]. $P(Z \le 3/4 | Y \le 1/4)$.

4. Insurance claims arrive at an insurance company according to a Poisson process with rate λ. The amount of each claim has an exponential distribution with rate μ independently of times and amounts of all other claims. Let X_t denote the accumulated total of claims between time 0 and time t. Find simple formulae for
[**a** (3 points)]. $E(X_t)$.
[**b** (5 points)]. $E(X_t^2)$.
[**c** (5 points)]. $SD(X_t)$.
[**d** (7 points)]. $Corr(X_t, X_S)$ for $t < s$.

5. Suppose X and Y are standard normal variables. Find an expression for $P(X + 2Y \le 3)$ in terms of the standard normal distribution function Φ:
[**a** (10 points)] in case X and Y are independent.
[**b** (10 points)] in case X and Y have bivariate normal distribution with correlation 1/2.

6.
[**a** (10 points)]. Let X and Y be two random variables taking two values 0 and 1. Show that if

$$P[X = 0, Y = 0] = P[X = 0]P[Y = 0]$$

then X and Y are independent.
[**b** (10 points)]. Find two random variables X and Y taking the values $0, 1$ and 2 such

$$P[X = 0, Y = 0] = P[X = 0]P[Y = 0]$$

but X and Y are *not* independent.