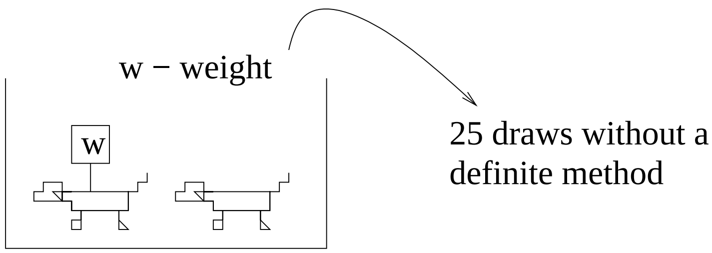
Lecture 7: Informal Hypothesis Testing

An example:

Lab with hundreds of mice



$\mu = 30$g $\qquad\qquad \bar{X}_n = 33$g
SD $= \sigma = 5$g $\qquad\qquad s = 7$g

Q: Was the sampling method SRS? or is 33g too far above average for that?

- Do we expect $\bar{X}_n \equiv 30$ if SRS is done?
- If we expect $\bar{X}_n$ to be different from $\mu = 30$, how much difference is "reasonable"?

1. Calculate the difference:

$$\bar{X}_n - \mu = 33 - 30 = 3\text{g}$$

   Is 3g too big a difference to expect from SRS?

2.

$$\frac{\bar{X}_n - \mu}{\mathrm{SD}(\bar{X}_n)} \sim \frac{\bar{X}_n - \mu}{\frac{1}{\sqrt{n}}\sigma} \cong \frac{\bar{X}_n - \mu}{\frac{1}{\sqrt{n}}s} = \frac{3}{\frac{1}{\sqrt{25}}7} = 2.14$$

3. Assessing the "extremeness" of 2.14:

   Fact: Weights follow a normal curve. Assume the population follow a Normal curve $\mathrm{N}(\mu, \sigma^2)$.

$$\frac{\bar{X}_n - \mu}{\frac{1}{\sqrt{n}}s} = t \sim t_{n-1} \text{ (if } \sigma \text{ is not available)}$$

   Students t-curve is also bell-shaped, but has a fatter tail than the Normal curve. In this example, $n - 1 = 24$.

$t = 2.06 \rightarrow 2.5\%$
$t = 2.49 \rightarrow 1\%$
$t = 2.79 \rightarrow 0.5\%$

$2.14 \rightarrow$ chance is less than 2.5%, but larger than 1% to observe 2.14 if SRS was used.

Is 2.5% small enough? Convention: 1% or 5% cut-offs

Summary:

Assume SRS from a known population:

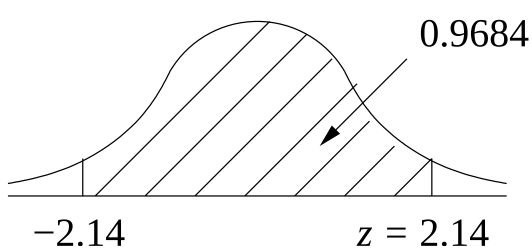
1. Calculate difference: $\bar{X}_n - \mu$
2. Standardize the difference: $\frac{\bar{X}_n - \mu}{\frac{1}{\sqrt{n}} s} = t_{n-1}$
3. Need sampling distribution of t_{n-1} to decide on the "extremeness" of observed t_{n-1}.
4. Small tail probability means "unlikely" "SRS with known population" holds.

What if the lab mice were over-fed so that their weight distribution is not normal?

Luckily, for "n large",

$$t_{n-1} = \frac{\bar{X}_n - \mu}{\frac{1}{\sqrt{n}} s} \sim \mathrm{N}(0, 1)$$

Looking up Normal table with $z = 2.14$ gives us: $\frac{1-0.9684}{2} = 0.0158$



1.58% - not too different from the t-table.

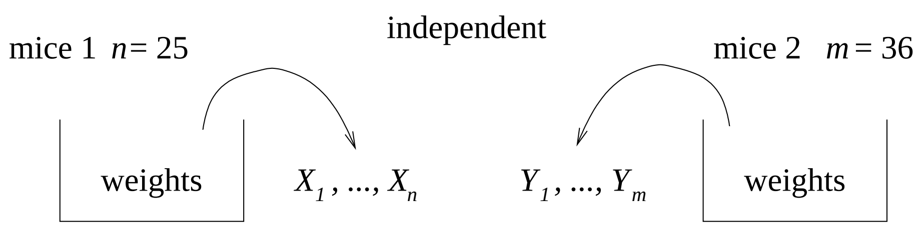
CLT: Central Limit Theorem (Very important probability result)

- It explains why the normal distribution is a good model, in addition to "normal" measurements on humans and animals.

- It says "small independent disturbances add up to display a normal distribution".
- e.g. measurement error in labs, "white noise" in deep space communication.
- Ref: Handout from Pitman
- The more symmetric the "box", the faster for the normal curve to emerge ...

Two Sample Case

Now the lab mice have been put into two samples.



$\bar{X}_n = 33$g
$s_1 = 7$g

$\bar{Y}_m = 35$g
$s_2 = 6$g

Two feeding methods...

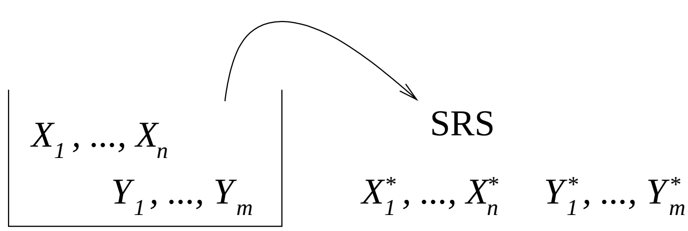
Q: Are the 2 feeding methods the same on average?

One way to formulate this question into mathematical terms: $\mu_1 = \mu_2$?

Observation: Even if $\mu_1 = \mu_2$, $\bar{X}_n \neq \bar{Y}_n$ most likely.

3a: Permutation Test

Assume $X_1, \ldots, X_n, Y_1, \ldots, Y_m$ exchangeable.



$\Rightarrow t^*$, repeat and get many values of t^* and tabulate.

3b: Assume $\sigma_1 = \sigma_2 = \sigma$, Normal population. Pooled estimate of the variance σ^2 is:

$$s^2 = \frac{(n-1)s_1^2 + (m-1)s_2^2}{n+m-2}$$

$$t = \frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\frac{s^2}{n} + \frac{s^2}{m}}} \sim t_{n+m-2}$$

1. Calculate the sample mean difference: $\bar{X}_n - \bar{Y}_m = 2$g
2. Standardize: need SD$(\bar{X}_n - \bar{Y}_m)$

$$\begin{aligned}
\text{Var}(\bar{X}_n - \bar{Y}_m) &= \text{Var}(\bar{X}_n) + \text{Var}(\bar{Y}_m) \\
&= \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m} \\
\text{SD}(\bar{X}_n - \bar{Y}_m) &= \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}} \\
&\cong \sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}
\end{aligned}$$

$$t = \frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}} = \frac{2}{\sqrt{\frac{49}{25} + \frac{36}{36}}} = 1.16$$

3. Assessing the "extremeness" of $t = 1.16$

3c: n, m large, CLT holds for $\bar{X}_n - \bar{Y}_m$, i.e. $t \cong$ N(0,1)

4. Chances of seeing something more extreme.

- 3a. Need computer help
- 3b. Assume $\sigma_1 = \sigma_2$, normal population.

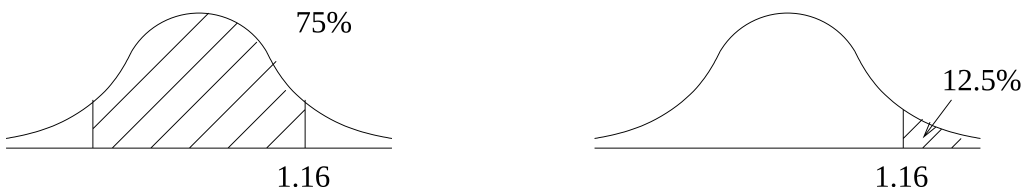
$$s^2 = \frac{24 \times 7^2 + 35 \times 6^2}{25 + 36 - 2} = 41.3$$

$$t = \frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\frac{41.3}{25} + \frac{41.3}{36}}} = \frac{2}{1.67} = 1.20.$$

$n + m - 2 = 25 + 36 - 2 = 59$ – large. $t_{59} \cong$ N(0,1) so look up normal table.

- 3c. Suppose the weights are not too asymmetric . . . CLT holds.

$$t = \frac{\bar{X}_n - \bar{Y}_m}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}} = \frac{2}{1.72} = 1.16$$



Assumptions matter a lot!