

Homework Assignment 6 due Thursday 5/9

1. We wish to test the null hypothesis H_0 that a given die is fair ($p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = 1/6$) against the alternative H_1 , that the die is biased in the following way: $p_1 = 1/5, p_6 = 1/10, p_2 = p_3 = p_4 = p_5 = p_6 = 7/40$. It is rolled 10 times.

- (i) Calculate the joint distribution of (n_1, n_6) under both H_0 and H_1 , where n_1 (respectively n_6) is the number of 1s (respectively 6s) among the 10 rolls.

We now plan to carry out a test of H_0 with alternative H_1 , on the basis of the outcome of the 10 rolls. Three statistics are under consideration to test this null hypothesis: the number n_1 of 1s; the number n_6 of 6s; and the likelihood ratio statistic.

- (ii) For each of these three statistics, determine a cut-off defining a rejection region which has a Type 1 error as close to 5% as is achievable.
 - (iii) Using the cut-offs determined in (i), calculate the power of each test, that is, the probability of rejecting the null hypothesis H_0 when the alternative H_1 is true.
2. (Continuation of 1) Suppose that the prior probability of the die being fair is 0.8 and that of it being biased in that particular way is 0.2. The die is rolled 10 times and $n_1 = 4, n_6 = 0$ is obtained. Calculate the posterior probability that the die is fair given these data. What function of the data goes into this calculation?
 3. Suppose 15 rats are used in a biomedical study where the rats are injected with cancer cells and given a cancer drug that is designed to increase their survival rate. The survival times, in months, are 14,17,27,18,12,8,22,13,19, and 12. Assume that the exponential distribution applies. Give a maximum likelihood estimate of mean survival.