Homework Assignment 5 due Tuesday 5/6

1. Suppose a particular kind of atom has a half-life of 1 year. Find:
   (a) the probability that an atom of this type survives at least 5 years;
   (b) the time at which the expected number of atom is 10% of the original;
   (c) if there are 1024 atoms present initially, the time at which the expected number of atoms remaining is one;
   (d) the chance that in fact none of the 1024 original atoms remains after the time calculated in (c).

2. Suppose component lifetimes are exponentially distributed with mean 10 hours. Find:
   (a) the probability that a component survives 20 hours;
   (b) the median component lifetime;
   (c) the SD of component liftime;
   (d) the probability that the average liftime of 100 independent components exceeds 11 hours;
   (e) the probability that the average liftime of 2 independent components exceeds 11 hours;

3. A group of 10 people agree to meet for lunch at a cafe between 12 noon and 12:15 p.m. Assume that each person arrives at the cafe at a time uniformly distributed between noon and 12:15 p.m., and that the arrival times are independent of each other.
   (a) Jack and Jill are two members of the group. Find the probability that Jack arrives at least two minutes before Jill.
   (a) Find the probability of the event that the first of the 10 persons to arrive does so by 12:05 p.m., and the last person arrives after 12:10 p.m.

4. Suppose that (X, Y) is uniformly distributed over the region
$$\{(x, y) : 0 < |y| < x < 1\}.$$
   Find:
   (a) the joint density of (X, Y);
   (b) the marginal densities $f_X(x)$ and $f_Y(y)$.
   (c) Are X and Y independent?
   (d) Find $E(X)$ and $E(Y)$.

5. For random variables X and Y with joint density function

$$f(x,y) = 6\,e^{-2x-3y} \qquad (x,y>0)$$

and $f(x,y) = 0$ otherwise, find

   (a) $P(X \le x, Y \le Y)$;

   (b) $f_X(x)$;

   (c) $f_Y(y)$;

   (d) Are X and Y independent? Give a reason for your answer.

6. Suppose the true weight of a standard weight is 10 grams. It is weighed twice independently. Suppose that the first measurement is a normal random variable X with $E(X) = 10\,g$ and $SD(X) = 0.2\,g$, and that the second measurement is a normal random variable Y with $E(Y) = 10\,g$ and $SD(Y) = 0.2\,g$.

   (a) Compute the probability that the second measurement is closer to $10\,g$ than the first measurement.

   (b) Compute the probability that the second measurement is smaller than the first, but not by more than $0.2\,g$.