

Homework Assignment 1

1. Sampling with and without replacement

Sampling with replacement

A box contains tickets marked 1, 2, ..., 10. A ticket is drawn at random from the box. Then this ticket is replaced in the box and a second ticket is drawn at random. Find the probabilities of the following events:

- (a) the first ticket drawn is number 1 and the second ticket is number 2.
- (b) the numbers on the two tickets are consecutive integers, meaning the first number drawn is one less than the second number drawn.
- (c) the second number drawn is bigger than the first number drawn.

Sampling without replacement

- (d) Repeat (a) through (c) assuming instead that the first ticket drawn is not replaced, so the second ticket drawn must be different from the first.

2. Suppose a deck of 52 cards is shuffled and the top two cards are dealt.

- (a) How many ordered pairs of cards could possibly result as outcomes?

Assuming each of these pairs has the same chance, calculate:

- (b) the chance that the first card is an ace;
- (c) the chance that the second card is an ace (explain your answer by a symmetry argument as well as by counting);
- (d) the chance that both cards are aces;
- (e) the chance of at least one among the two cards.

3. The odds against an event occurring are 10 to 1. What is the chance of the event? What if the odds were 5 to 1 against?

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4. Electrical components

Suppose there are two electrical components. The chance that the first component fails is 10%. If the first component fails, the chance that the second component fails is 20%. But if the first component works, the chance that the second component fails is 5%.

Calculate the probabilities of the following events:

- (a) at least one of the components works;
- (b) exactly one of the components works;
- (c) the second component works.

5. Distribution of sexes.

Assume that boys and girls are born with equal frequencies, not quite true, but the difference doesn't matter for this problem. Consider families of exactly two children. Given that a family has a boy, what is the probability that both children are boys?

6. False positives.

Suppose that 1% of the population has a particular disease. There is a test on a blood sample which yields one of two results, positive or negative. It is found that 95% of the people having the disease produce a positive result. But 2% of the people without the disease will also produce a positive result. What is the probability that a person randomly chosen from the population will have the disease, given that the person's test was positive?