Warm-up exercises for individual practice (with solutions).

You have a basket with an eggplant, a carrot, a banana, a strawberry, a lettuce, a red bell pepper, zucchini, and cucumber.

1. You pick one at random.
(i) What is the chance to get a fruit?
(ii) What is the probability to pick a veggie?
(iii) What is the chance to get a green item?
(iv) What is the probability to get a green item or a fruit?
(v) What is the probability to pick a red item or a fruit?

2. You pick two at random, without replacing the first item.
(i) What is the chance that you pick a green item both times?
(ii) What is the probability that at least one of the item you picked is green?
(iii) What is the chance that a carrot is among the ones you picked?
The above exercises are helpful to understand the difference between mutually exclusive and independent, and to learn what this has to do with the addition rule and the multiplication rule. The last question is a slightly simpler version of the question from Monday in class: What is the probability that one of the 6 different numbers in a fortune cookie is 15?)

3. You pick three at random, without replacing the first or the second item.
(i) What is the chance to get at least two veggies?
(ii) What is the probability to get at most one fruit?
(iii) What is the probability to pick no veggie?
(iv) What is the probability to pick no fruit?

Recall: Addition rule

Notation: $P(A)$ means the probability that A happens. A and B means that the happen both. A or B means that at least one of them happens.
Addition rule (general form): Given two events A and B you have

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

A and B are called *mutually exclusive* if they cannot happen at the same time, that means A and B is impossible. In this case, the addition rule becomes a little shorter:

$$P(A \text{ or } B) = P(A) + P(B).$$

Solutions

1.(i) There are two fruits, a strawberry and a banana. By the definition of probability, you have to devide the number of fruits by the total number of items in the basket. This yields $P(\text{fruit}) = 2/8 = 1/4$.
(ii) Similarly to (i) you obtain $6/8 = 3/4$. Alternatively, you can you the opposite rule and get

$$P(\text{veggie}) = 1 - P(\text{fruit}) = 1 - 1/4 = 3/4.$$

(iii) There are three green items, a lettuce, zucchini, and cucumber. This yields $P(\text{green item}) = 3/8$.
(iv) There are three green items, a lettuce, zucchini, and cucumber. None of the fruits is green. Therefore the event to pick a green item and the event to pick a fruit are mutually exclusive. By the addition rule for mutually exclusive events we obtain This yields

$$P(\text{green item or fruit}) = P(\text{green item}) + P(\text{fruit}) = 3/8 + 2/8 = 5/8$$

(v) The question is similar to (iv), but the in this case we need to use the general addition rule. Since one of the fruit, the strawberry, is red, the event to pick a red item and the event to pick a fruit are not mutually exclusive. We obtain

$$P(\text{red item or fruit}) = P(\text{red item}) + P(\text{fruit}) - P(\text{red and fruit}) = 2/8 + 2/8 - 1/8 = 3/8$$

2.(i) Picking a green item both times means that the first and the second choice are green. Using independence we have

$$\begin{aligned} P(\text{first is green and second is green}) =& P(\text{first is green}) \cdot P(\text{second is green given first is green}) \\ =& \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28} \end{aligned}$$

(ii) Let us first compute the probability that we get a green item in exactly one of the draws. In a second step, we combine this with the result of (i). By addition rule,

$$\begin{aligned} P(\text{exactly one green}) =& P(\text{first green, second not}) + P(\text{second green, first not}) \\ =& \frac{3}{8} \cdot \frac{5}{7} + \frac{5}{8} \cdot \frac{3}{7} = \frac{15}{28} \end{aligned}$$

Note that we used independence, and that the two addends are equal. Again by addition rule we obtain

$$\begin{aligned} P(\text{at least one green}) =& P(\text{exactly one green}) + P(\text{both green}) \\ =& \frac{15}{28} + \frac{3}{28} = \frac{18}{28} = \frac{9}{14} \end{aligned}$$

(iii) Here are three different ways to solve the exercise:
(a) Let us first compute the number of ways to get a carrot by picket twice, without replacement. There is only one carrot in the basket. If you pick it first than you have still 7 possibilities for your second draw. If you second draw is a carrot then your first choice must have been one of the other 7 items. These two cases add up to 14 ways. The total number of possibilities you have drawing two items out of 8 is $8 \cdot 7$. Using the definition of probability, we obtain that the desired probability is $\frac{2 \cdot 7}{8 \cdot 7} = \frac{1}{4}$.
(b) Since there is only one carrot, you cannot get a carrot in both draws. The event of getting a carrot in the first draw and the event of getting a carrot in the second draw are mutually exclusive. By addition rule we obtain

$$\begin{aligned} P(\text{one of them is a carrot}) =& P(\text{the first is a carrot or the second is a carrot}) \\ =& P(\text{the first is a carrot}) + P(\text{the second is a carrot}) \\ =& 1/8 + 1/8 = 1/4. \end{aligned}$$

(c) Assign the numbers 1 to 8 to the items in the basket. 1 and 2 correspond to the first two draws. The probability that the carrot gets the number 1 or 2 is 2/8=1/4.
3.(i) We first introduce short notations: v means veggie, and f means fruit. vvf means the first one is a veggie, and the second one and the third one are fruits. Other triples are defined similarly. To compute the probablity in question we first use addition rule and then multiplication rule. Note that the events vvf, vfv, fvv, and vvv are mutually exclusive.

$$\begin{aligned} P(\text{at least 2 veggies}) &= P(\text{vvf or vfv or fvv or vvv}) \\ &= P(\text{vvf}) + P(\text{vfv}) + P(\text{fvv}) + P(\text{vvv}) \\ &= \frac{6 \cdot 5 \cdot 2}{8 \cdot 7 \cdot 6} + \frac{6 \cdot 2 \cdot 5}{8 \cdot 7 \cdot 6} + \frac{2 \cdot 6 \cdot 5}{8 \cdot 7 \cdot 6} + \frac{6 \cdot 5 \cdot 4}{8 \cdot 7 \cdot 6} \\ &= \frac{3 \cdot 6 \cdot 5 \cdot 2 + 6 \cdot 5 \cdot 4}{8 \cdot 7 \cdot 6} = \frac{3 \cdot 5 + 5 \cdot 2}{4 \cdot 7} = \frac{25}{28} \end{aligned}$$

(ii) This is just another formulation of question (i).
(iii) Impossible, since there are only two fruits. That means that the probability is 0.
(iv) By multiplication rule we obtain

$$P(\text{all three are veggies}) = \frac{6}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} = \frac{5}{14}.$$