Stat 248 – Handout 1 - Elementary Concepts and Methods - F07

Progression:
  Scientific question
  Experiment
    Measurements/data
  Probabilities defined?
  Statistical model   Inferences

Probabilities   P(A), P(B), …

Random variables   X, Y, …

c.d.f. $F(x) = Prob(X \le x)$

Function of r.v. $Z = h(X)$

Expected value

Discrete $\mu_Z = \Sigma\, h(x)\, Prob(X=x) = E(Z)$

Continuous $\int h(x)\, f(x)\, dx$, $f(x)dx = prob\{x<X<x+dx)$, dx small

Mean $\mu_X = E(X)$

Variance $\sigma_X^2 = \sigma_{XX} = E(Z),\ Z = (X - \mu_X)^2$

S.D. σ_X

Covariance $\sigma_{XY} = E(Z)$, $Z = (X - \mu_X)(Y - \mu_Y)$

Correlation $\rho_{XY} = \sigma_{XY} / \sigma_X \sigma_Y$

$-1 \le \rho_{XY} \le 1$

$\rho_{XY} = 0$ if X, Y independent $\rho_{XY} = +\text{-}1$ if $Y = \alpha + \beta X$

Distributions

Discrete
  Poisson $P(X=x) = \mu^x \exp(-\mu) / x!$, x = 0, 1, 2, …

Continuous
  Uniform $f(x) = 1/(B-A)$, $A < x < B$
  Exponential $\exp(-x)$, $0 < x < \infty$
  Normal/Gaussian $\varphi(z) = \exp(-z^2/2)/\sqrt{(2\pi)}$, $-\infty < z < \infty$

Least squares prediction

$\min_{\alpha,\beta} \; E(Y - \alpha - \beta X)^2$

$\mu_Y = \alpha + \beta \mu_X$

$\sigma_{YX} = \beta \sigma_{XX}$

$\min = \sigma_{YY} (1 - \rho_{YX}{}^2)$

Statistical inference

Sample $X_1, \ldots, X_n$ i.i.d.

Estimates

c.d.f. $\#\{x_i \le x\}/n$

μ_X x-bar

σ_{XX} $\Sigma (x_i - \text{x-bar})^2 /(n-1) = s_{XX}$

σ_{YX} $\Sigma (y_i - \text{y-bar}) (x_i - \text{x-bar}) /(n-1) = s_{YX}$

$\rho_{YX} = s_{YX} / s_X \; s_Y$

Least squares

$\min \Sigma (y_i - a - b x_i)^2$

y-bar = a + b x-bar

$s_{YX} = b \, s_{XX}$

Likelihood $L(\theta|x_1, \ldots, x_n) = \Pi f(x_i \mid \theta)$

Maximum likelihood estimation

$\max_\theta \; L(\theta|x_1, \ldots, x_n)$

Margin of error 2 s.e. (θ-hat)

Approximate 95% confidence interval θ-hat +- ME