TABLE 1
*Some max-type RDEs. Functions $g(\cdot)$ for which the RDE $X \stackrel{d}{=} g((\xi_i, X_i), i \geq 1)$ are discussed**

| Section | $g(\cdot)$ | Underlying model | Endog? | Comments |
|---|---|---|---|---|
| | $S = \mathbb{R}^+$ | | | |
| 4.2 | $\max_i (X_i + \xi_i)^+$ | Range of BRW | Yes | |
| 4.3 | $\min_i (X_i + \xi_i)^+$ | Algorithm for BRW range | Yes | |
| 4.6 | $\max_i (\xi_i - X_i)^+$ | Matching on GW tree | Yes | |
| 4.4 | $\xi_0 + \max_i (\xi_i X_i)$ | Discounted tree sums | Yes | $\xi_0 = 0$ reduces to BRW extremes |
| 4.4 | $\xi_0 + \min_i (\xi_i X_i)$ | Discounted tree sums | Yes | See (49) |
| 4.6 | $(\xi_0 - \sum_i X_i)^+$ | Independent subset GW tree | Yes | |
| 7.2 | $\sum_i (c - \xi_i + X_i)^+$ | Percolation of MSTs | Yes | Determines critical $c$ |
| 7.6 | See (98) | First passage percolation | Conj. $Y$ | Mean-field scaling analysis |
| | $S = \mathbb{R}$ | | | |
| 5 | $c + \max_i (X_i + \xi_i)$ | Extremes in BRW | No | $c$ specified by dist($\xi_i$) |
| 7.3 | $\min_i (\xi_i - X_i)$ | Mean-field minimal matching | Yes | |
| 7.4 | $\min_i^{[2]} (\xi_i - X_i)$ | Mean-field TSP | Conj. $Y$ | $\min^{[2]}$ denotes second smallest |
| | Other $S$ | | | |
| 6 | $\Phi(\min(X_1, X_2), \xi_0)$ | Frozen percolation on tree | Yes | $\Phi$ defined in Section 6 |
| 7.6 | See (96), (97), (98) | Mean-field scaling | Conj. $Y$ | $S = \mathbb{R}^2$ or $\mathbb{R}^3$ |

*Note $x^+ = \max(x, 0)$. For $S = \mathbb{R}$ a "max" problem is equivalent to a "min" problem by transforming $X$ to $-X$, but for $S = \mathbb{R}^+$ this does not work: the problems in Sections 4.2 and 4.3 are different. Typically the $(\xi_i)$ are either i.i.d. or are the successive points of a Poisson process on $(0, \infty)$. "Endogenous" refers to fundamental solution. Key to acronyms: BRW, branching random walk; GW, Galton–Watson; MST, minimal spanning tree; TSP, traveling salesman problem.

1.1.1. *Direct use.* Here is the prototype example of *direct use*, where the original question asks about a random variable $X$ and the distribution of $X$ itself satisfies an RDE.

EXAMPLE 1. Let $X$ be the total population in a Galton–Watson branching process where the number of offspring is distributed as $\xi$. In the case $\mathbb{E}\xi \leq 1$ [and $\mathbb{P}(\xi = 1) \neq 1$] it is well known that $X < \infty$ a.s., and then easy to check that dist($X$) is the unique solution of the RDE

$$X \stackrel{d}{=} 1 + \sum_{i=1}^{\xi} X_i \qquad (S = \mathbb{Z}^+).$$

We will see other direct uses in Proposition 25 and in the examples in Section 4.4.

1.1.2. *Indirect use.* The simplest kind of indirect use is where the quantity of interest can be written in terms of known quantities and some other quantity