Statistical Analysis of Nuel Tournaments

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Introduction

Truel, *Quadruel*, and *Nuel* is simply an extension of a duel tournament, which involves more than two people (*Pirates of Caribbean* three-way sword fight serves as an appropriate visual representation). Nuel tournament is one of classic statistics paradoxes that illustrate a fitter/better competitor in a multi-entry survival type competition does not necessarily possess significantly higher chance of survival than others.

Objectives

A Nuel tournament may involve wide spectrum of variables that leads to diverse game design, such as number of players, shooting sequence, varying marksmanship, randomness in shooting. In this project, I was particularly interested in observing non-simultaneous, sequential Nuel tournaments.

Objectives of this project is to:

- Gain a general mathematical understanding of Nuel tournaments, by constructing Markov Chains and running simulations. Then, the simulation results will be compared to long-term transition probabilities presented in Markov Chains.
- Gain a behavioral understanding through programming and conducting experiments to other people. The purpose of conducting experiments is to recognize any interesting patterns among game participants' responses, and to compare such responses against theoretical probabilities computed by Markov Chain.
- Perform theoretical analysis in order to interpret various Nuel tournaments. This objective may involve both mathematical analysis of Markov Chains and a sensitivity analysis involving varying levels of each players' marksmanship.

Simulation and experiment is coded in Java programming, Markov Chains are generated from MATLAB, and experiment results are presented in R.

Method Design

Both the simulation and the experiment comprises of four different game sets. Each game set involves different accuracy level of shooters (please refer to figure 1), and is comprised of three different rounds respectively with 3, 4, and 5 players. For each game round, the shooting sequence is always set to be player 1 → 2 → 3 … → 1, etc. Each player shoots one at a time, and simultaneous shooting was not allowed in this game. Each game round is played until there is only one survivor left.

Theoretical Method Design

Markov Chain matrix was constructed for a Nuel, where:

State i	Representation
1	N players alive
$(2, 2+\binom{N}{1})$	(N-1) players alive
$(2+\binom{N}{1}, 2+\binom{N}{1}+\binom{N}{2})$	(N-2) players alive
...	...
$(2+\sum_{i=1}^{N-2}\binom{N}{i}, 2+\sum_{i=1}^{N-1}\binom{N}{i})$	1 player alive

To obtain theoretical probabilities for each player's chance of survival, I computed long-term probability

$$\pi_j = \lim_{n\to\infty} p_{ij}^{(n)}$$

for all states j where there is only one survivor.

Simulation: For each set, a total of 750,000 game round simulation was run (i.e. 250,000 game rounds with three different variations in number of players). Each simulation assumes to be i.i.d (identically and independently distributed), and random numbers were generated uniformly.

Experimental Method Design

In each experimental session, participants were asked to play four different types of Nuel games. Total of 27 college students at UC Berkeley were asked to participate in the experiment. For each game type, each participant were asked to choose the player that each participant believed to have the highest chance of survival. Throughout the game, participants were allowed to switch their choices to a different player if they desired so.

Set 1: $p_1=\frac{1}{2}, p_2=\frac{1}{3}, p_3=\frac{1}{4}, \dots$

Set 2: $p_1=\frac{1}{4}, p_2=\frac{1}{3}, p_3=\frac{1}{2}, \dots$

Set 3: $p_1=\frac{N}{N}, p_2=\frac{N-1}{N}, p_3=\frac{N-2}{N}, \dots$

Set 4: $p_1=\frac{1}{N}, p_2=\frac{2}{N}, p_3=\frac{3}{N}, \dots$

Figure. 1 A table enlisting four different sets of players' accuracy level in the experiment. Accuracy level of each player i is denoted with p_i

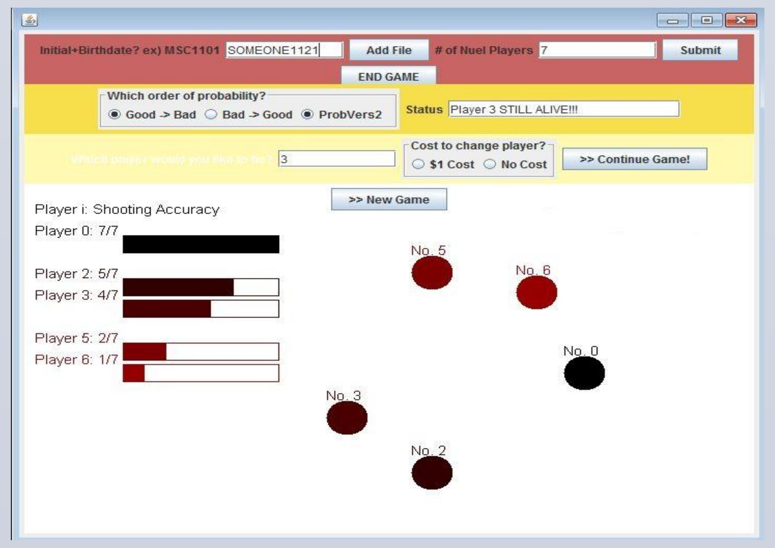
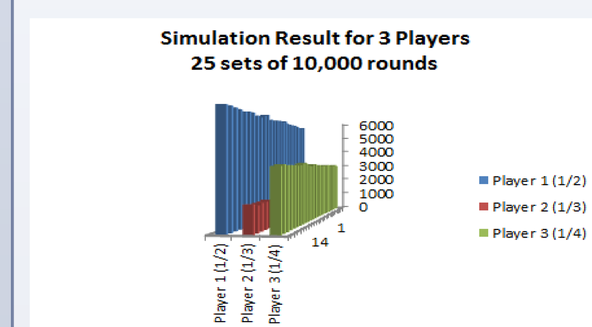


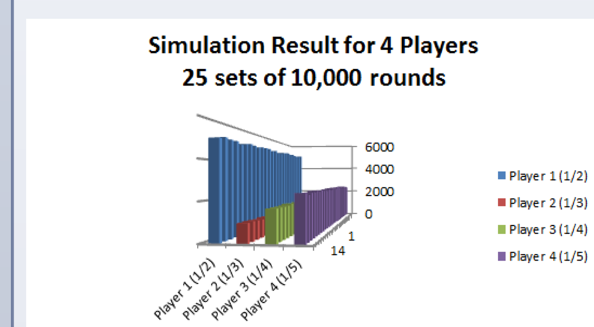
Figure. 2 Demonstration of a Nuel Experiment Program designed on Java Eclipse platform.

Theoretical Results

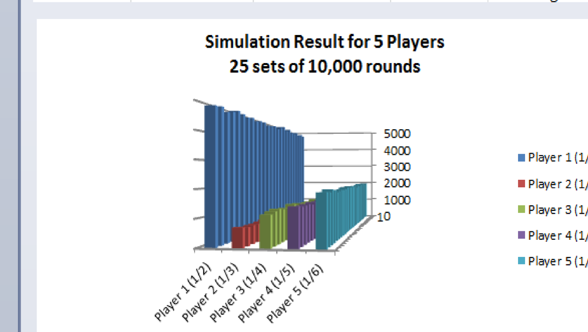
Set 1: $p_1=\frac{1}{2}, p_2=\frac{1}{3}, p_3=\frac{1}{4}, \dots$



Player 1	Player 2	Player 3	
5671.72	1295.12	3033.16	Simulation Avg
5667	1296	3037	Theoretical Avg
0.083289218	0.067901235	0.126440566	Percentage Error



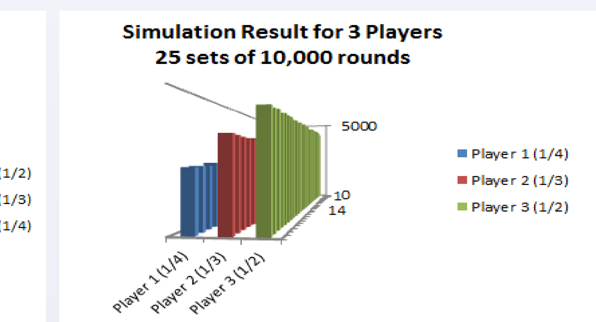
Player 1	Player 2	Player 3	Player 4	
5008	947.72	1671.04	2373.24	Simulation Avg
5203	950	1723	2124	Theoretical Avg
3.747837786	0.24	3.015670342	11.73446328	Percentage Error



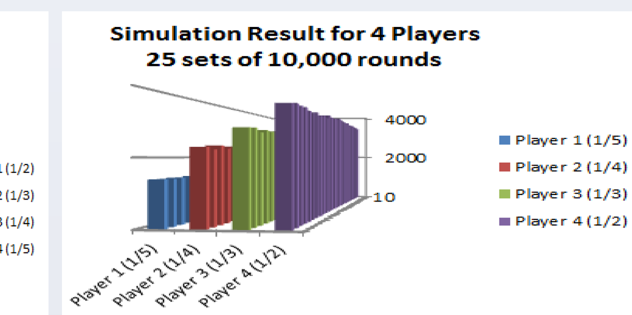
Player 1	Player 2	Player 3	Player 4	Player 5	
4816.2	688	1216.04	1377.48	1902.28	Simulation Avg
4804	666	1195	1361	1832	Theoretical Avg
0.253955	3.303303	1.760669	1.210874	3.836245	Percentage Error

Player 1 seems to have the highest chance of survival, followed by player 5, 4, 3, and 2. Simulation results and values derived from MC doesn't seem to be significantly different.

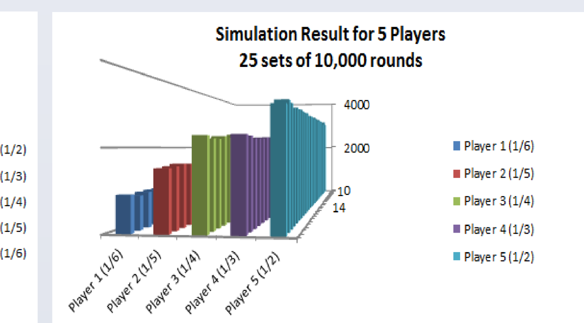
Set 2: $p_1=\frac{1}{4}, p_2=\frac{1}{3}, p_3=\frac{1}{2}, \dots$



Player 1	Player 2	Player 3	
2336.8	3339.64	4323.56	Simulation Avg
2333	3333	4333	Theoretical Avg
0.162880411	0.199219922	0.217862913	Percentage Error



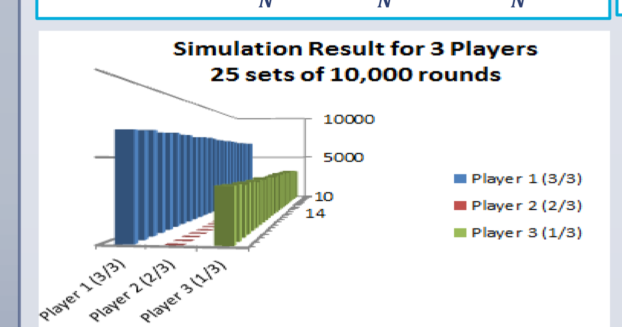
Player 1	Player 2	Player 3	Player 4	
1391.64	2319.52	2796.36	3492.48	Simulation Avg
1418	2310	3063	3210	Theoretical Avg
1.858956	0.412121	8.705191	8.8	Percentage Error



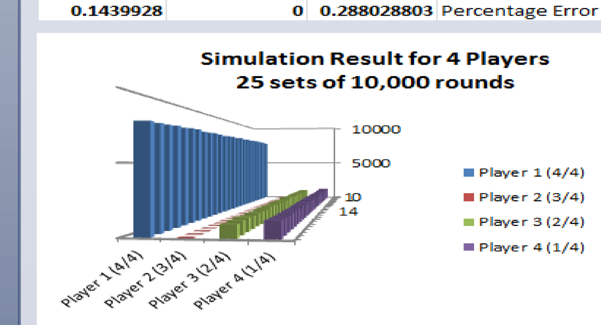
Player 1	Player 2	Player 3	Player 4	Player 5	
881.64	1524.68	2285.4	2257.44	3050.84	Simulation Avg
822	1354	2143	2154	2653	Theoretical Avg
7.255474453	12.605613	6.644890341	4.802228412	14.99585375	Percentage Error

Player 5 seems to have the highest chance of survival, followed by players 4 and 3 (on tie), then 2 and 1. Simulation results seem to deviate a bit from theoretical values.

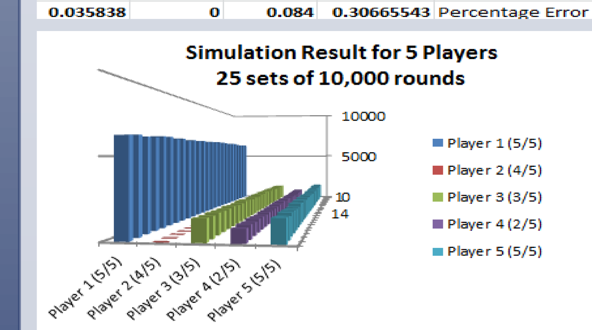
Set 3: $p_1=\frac{N}{N}, p_2=\frac{N-1}{N}, p_3=\frac{N-2}{N}, \dots$



Player 1	Player 2	Player 3	
6676.6	0	3323.4	Simulation Avg
6667	0	3333	Theoretical Avg
0.1439928	0	0.288028803	Percentage Error



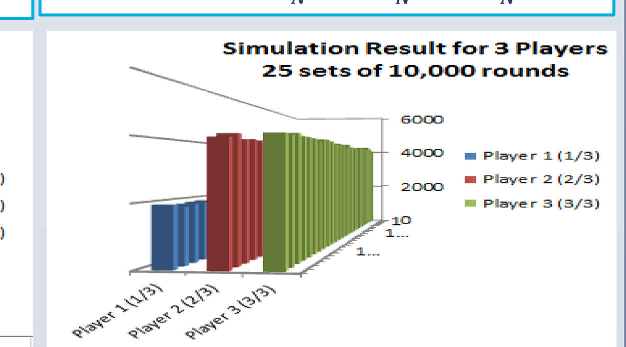
Player 1	Player 2	Player 3	Player 4	
7810.2	0	999.16	1190.64	Simulation Avg
7813	0	1000	1187	Theoretical Avg
0.035838	0	0.084	0.30665543	Percentage Error



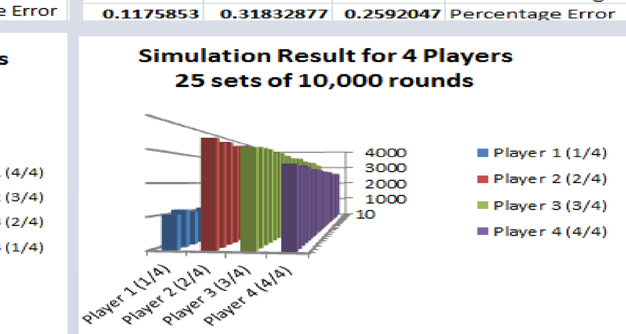
Player 1	Player 2	Player 3	Player 4	Player 5	
6313.04	0	1377.84	855.84	1453.28	Simulation Avg
6305	0	1385	859	1450	Theoretical Avg
0.127518	0	0.516968	0.36787	0.226207	Percentage Error

Player 1 seems to have the highest chance of survival, followed by player 5, 3, 4, and 2. Simulation results and values derived from MC doesn't seem to be significantly different.

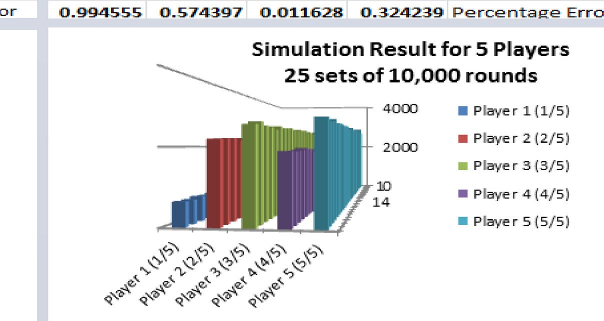
Set 4: $p_1=\frac{1}{N}, p_2=\frac{2}{N}, p_3=\frac{3}{N}, \dots$



Player 1	Player 2	Player 3	
1907.24	4008.2	4084.56	Simulation Avg
1905	4021	4074	Theoretical Avg
0.1175853	0.31832877	0.2592047	Percentage Error



Player 1	Player 2	Player 3	Player 4	
1091.04	3291.8	3096.36	2520.8	Simulation Avg
1102	3273	3096	2529	Theoretical Avg
0.994555	0.574397	0.011628	0.324239	Percentage Error

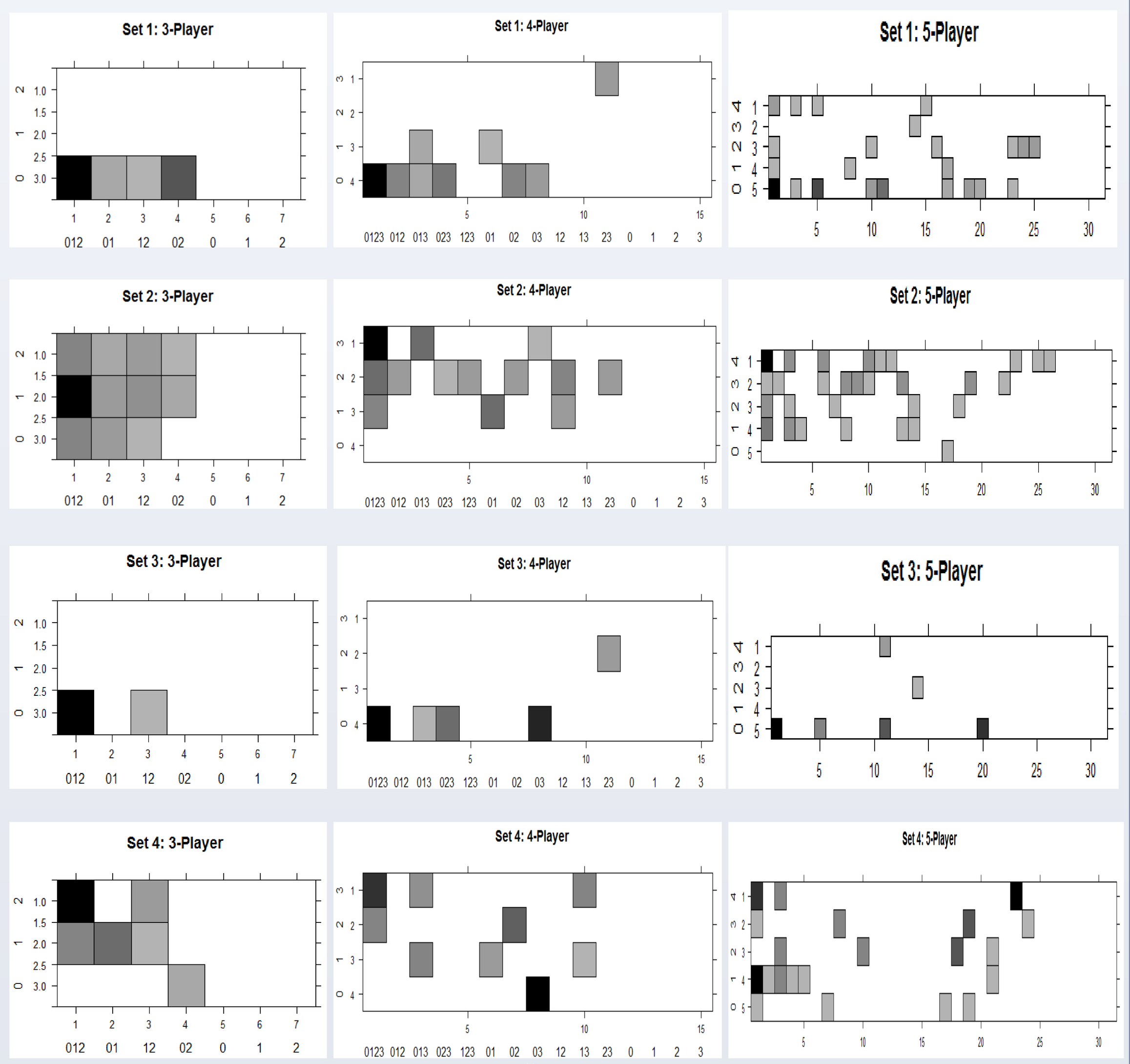


Player 1	Player 2	Player 3	Player 4	Player 5	
649.2	2189.72	2576.76	1891.96	2692.36	Simulation Avg
628	1769	2581	1569	2351	Theoretical Avg
3.375796	23.78293	0.164277	20.58381	14.51978	Percentage Error

This set of result is quite interesting, as a player with perfect shots does not show a dominant chance of survival over other players.

Experimental Results

X-axis represent each state (i.e. list of alive players) in Markov Chain, and y-axis represent participant's player choices based on each Markov State.



Conclusions

Theoretical:

It turns out that the simulation results match quite closely with the results derived from MC long-transition probabilities. One might be interested in running simulations under different distributions, such as normal or gamma dist. A further sensitivity analysis is required for further analysis.

Experimental:

I was able to observe variability patterns. Three significant factors that cause variability in participants' choices are: 1) number of players, 2) Presence of a player with perfect shot, 3) Sequence (i.e. choices vary when weaker players shoot first). Overall, a significant conclusion is that participants *do* tend to choose players primarily based on accuracy level and not the chance of survival. A further correlation analysis is required to recognize any correlation patterns.

References

[1] P. Amengual and R. Toral, "Truels, or Survival of the Weakest," IEEE Transactions Computing in Science & Engineering, September/October 2006, pp.88-89.

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