Statistical Analysis of Nuel Tournaments

Department of Statistics
University of California, Berkeley

MoonSoo Choi
Department of Industrial Engineering & Operations Research

Under Guidance of

Professor. David Aldous
Department of Statistics

First Draft Written on:

5/18/2013

This paper explores and presents visual analysis of N-uel tournaments and cognition/perception analysis of N-uel game players decision-making process under various visual representations.

Table of Contents

I. Abstract

Among numerous classical statistics paradoxes, N-uel (for $N \ \mathbb{N} \setminus \{1\}$, because N-uel where $N = 1$ is clearly not that interesting!) problem shows that the fittest of all competitors does not necessarily win this competition. *Truel, Quadruel,* and *Nuel* is simply an extension of a duel tournament, which involves more than two people (*Pirates of Caribbean* three-way sword fight serves as an appropriate visual representation). In addition, each N-uel player actually may possess different winning maximization strategy, since the Nuel tournament is known to illustrate that a fitter/better competitor in a multi-entry survival type competition does not necessarily possess significantly higher chance of survival than others. It is my desire to see this process: I want to visualize the N-uel process as a third-person observer, and if a person was playing this as a video game (which I plan on developing via Java Programming), I also want to visualize the winning strategies each video game participant follows.

II. Introduction

A Nuel tournament may involve wide spectrum of variables that leads to diverse game design, such as number of players, shooting sequence, varying marksmanship, randomness in shooting. In this project, I was particularly interested in observing non-simultaneous, sequential Nuel tournaments.

Objectives of this project are to:

- Gain a general mathematical understanding of Nuel tournaments, by constructing Markov Chains and running simulations. Then, the simulation results will be compared to long-term transition probabilities presented in Markov Chains.
- Gain a behavioral understanding through programming and conducting experiments to other people. The purpose of conducting experiments is to recognize any interesting patterns among game participants' responses, and to compare such responses against theoretical probabilities computed by Markov Chain.
- Perform theoretical analysis in order to interpret various Nuel tournaments. This objective may involve both mathematical analysis of Markov Chains and a sensitivity analysis involving varying levels of each players' marksmanship.

Simulation and experiment is coded in Java programming, Markov Chains are generated from MATLAB, and experiment results are presented in R.

III. Probabilistic Model Design

i. Variables, Parameters, and Assumptions

As the objective of this research is to merely examine the general scope of the Nuel Tournament, I decided to observe Nuel Tournaments with few variables in interest, which includes the following:

a) Number of Players (i.e. 3, 4, and 5)
   - ❖ Integer Variable $\in \{3, 4, 5\}$

b) Sequence of shooting orders
   - ❖ Boolean Variable $\in \{0, 1\}$
     i. **1** if the strongest shoots first, then the shooting progresses in order of decreasing marksmanship
     ii. **0** if the weakest shoots first, then the shooting progresses in order of increasing marksmanship

c) Presence of a perfect player (i.e. the marksmanship of the player is 100%)
   - ❖ Boolean Variable $\in \{0, 1\}$
     i. **1** if a perfect-shot player present in the tournament
     ii. **0** otherwise

d) Player's marksmanship (i.e. accuracy level – note that this does not imply survivability level!)
   - ❖ Real Variable $\in [0, 1]$

Each of the variable above will be denoted respectively with the following notations throughout this paper: ***n, s_n, X,*** and ***p_i***. Now I will discuss the parameters and the game design for the Nuel Tournament.

Parameter I) Each player with marksmanship level of p_i is assumed to be i.i.d

Parameter II) Each player with marksmanship level of p_i is assumed to be uniformly distributed.

Parameter III) Every tournament will be a "hunger-game" survival-style – that is, players in each tournament will continue to play the rounds until there is only one survivor.

Parameter IV) 1 round will consist of each shooter attempting to shoot the shooter next to him (that is, shooting the shooter that possesses the next highest/lowest marksmanship within the tournament)

IV – i) The last person will attempt to shoot the first shooter of the tournament, and such attempt will mark the end of the round (i.e. player ***n*** in a ***n***

IV – ii) As noted in parameter III, the rounds will continue until there is only survivor left in the game

With the assumptions and parameters stated above, I ultimately came up with 12 following sets of variables in this research (please refer to the variables above as needed):

(***n*, *s*$_n$, *X*,** and ***p**$_i$*)

Set I – III: (*n*, 1, 0, ($p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{4}$, ...)) where *n* = 3, 4, 5 respectively for sets I – III.

→ In other words, from strongest to weakest order without the presence of perfect player

Set IV-VI: (*n*, 0, 0, ($p_1 = \frac{1}{n+1}$, $p_2 = \frac{1}{n}$, $p_3 = \frac{1}{n-1}$, ...)) where *n* = 3, 4, 5 respectively for sets IV – VI.

→ In other words, from weakest to strongest order without the presence of perfect player

Set VII – IX: (*n*, 1, 1, ($p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{4}$, ...)) where *n* = 3, 4, 5 respectively for sets VII – IX.

→ In other words, from strongest to weakest order with the presence of perfect player

Set X – XII: (*n*, 0, 1, ($p_1 = \frac{1}{n+1}$, $p_2 = \frac{1}{n}$, $p_3 = \frac{1}{n-1}$, ...)) where *n* = 3, 4, 5 respectively for sets X – XII.

→ In other words, from weakest to strongest order with the presence of perfect player

ii. Model

With above variables and parameters in mind, I constructed a Markov Chain as following, with the description of the states as explained in the following figure:

State i	Representation
1	N players alive
$(2, 2 + \binom{N}{1})$	(N-1) players alive
$(2 + \binom{N}{1}, 2 + \binom{N}{1} + \binom{N}{2})$	(N-2) players alive
...	...
$(2 + \sum_{i=1}^{N-2} \binom{N}{i}, 2 + \sum_{i=1}^{N-1} \binom{N}{i})$	1 player alive

First state in the Markov Chain denotes that all the players are alive in the tournament. Now, assume that the tournament involves 4 players. Next, the states, 2, 3, 4, 5 will denote that player 1, 2, 3, and 4 is out of the tournament, respectively. Then, the states, 6 … 11 will denote the survival of two players, etc.

Figure 1. Markov Chain States

With the Markov states defined as above, to obtain theoretical probabilities for each player's chance of survival, I computed the long-term transition probability, for all states *j* where there exists only one survivor. Note that the transitional probability $p_{ij}^{(n)}$ denotes a matrix entry (*i, j*) in n-time Cartesian product of a Markov Matrix.

$$\pi_j = \lim_{n \to \infty} p_{ij}^{(n)}$$

Each Markov Chain was generated via MATLAB, where a function to generate Markov matrices with respect to each player's marksmanship was implemented. Markov Chain Matrix generating functions may found at the Appendix section of this paper.

Here is a sample long-term probability for three-shooter tournament with equal level of marksmanship. As one may observe from the figure, while the marksmanship of three player is equally distributed with 0.5 for each player, the first shooter possesses higher level of survivability likelihood over other two shooters. The numbers, 0.42857, indicate that a player with such survivability will survive approximately 4286 out of 10,000 times.

Complexity and exponential growth in size of Markov Chain Matrix size (i.e. a tournament with n number of players will involve 2^{n-1} states, which is extremely difficult for a computer to run a simulation for strategy optimization) motivated me to work on potential matrix decomposition to construct a potential strategy optimization model. If matrix decomposition is possible, this may also lead to a formulation of an optimization programming problem. This work is currently in progress.

iii. Simulation

In order to verify that the transitional probability values generated via Markov Chain is indeed valid, I conducted numerous simulations on Java to verify the results. As mentioned earlier in the paper, I explored total of twelve different types of Nuel Tournaments in the research. For each of the tournament type, I ran 10,000 simulations and computed the probability of survivability by simple calculation below:

$$\text{(Probability of Survivability)} = \frac{(\#\ of\ Tournaments\ Survived)}{10000\ Tournaments}$$

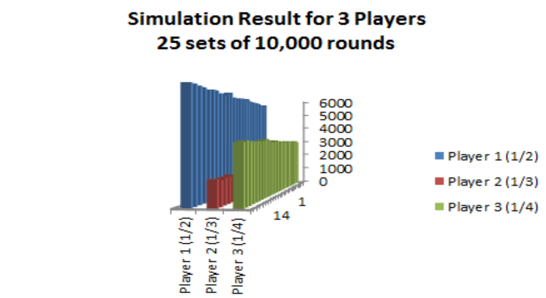
Then, for each player i, the above probability is compared against each of the long-term transitional probability (from state 1 (i.e. where all N players are alive) to state j)

0	0	0	0	0.42857	0.19048	0.38095
0	0	0	0	0.33333	0.66667	0
0	0	0	0	0.33333	0	0.66667
0	0	0	0	0	0.33333	0.66667
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1

Figure 2. Long-Term Transition Probability

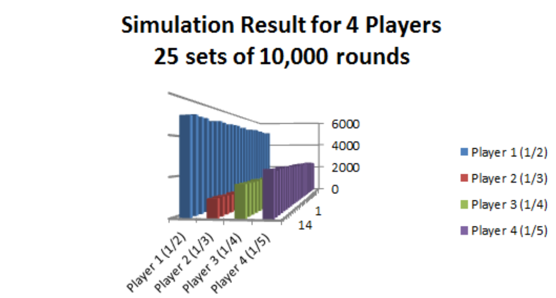
$$\pi_j = \lim_{n\to\infty} p_{1,j}^{(n)} \ , where\ \forall\ j = \{\forall i = 1,2,\ldots,N\text{: } set\ of\ states\ where\ player\ i\ is\ alive\}$$

Each of the simulation is run with the assumptions and parameters mentioned above, and a random number between 0 and 1 is generated uniformly in order to determine whether each player successfully murders the target (i.e. for each p_i the target is dead if $P\ \{p_i \leq x\}$, where x denotes the value randomly generated in the simulation. Resulting graphs for all simulations are displayed on the next page:



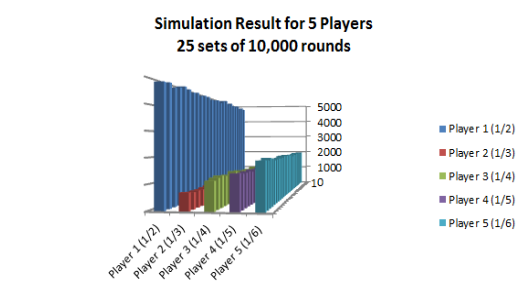
Player 1	Player 2	Player 3	
5671.72	1295.12	3033.16	Simulation Avg
5667	1296	3037	Theoretical Avg
0.083289218	0.067901235	0.126440566	Percentage Error

Figure 3-1. Set I Simulation Results



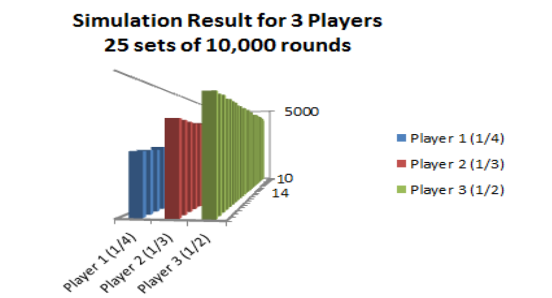
Player 1	Player 2	Player 3	Player 4	
5008	947.72	1671.04	2373.24	Simulation Avg
5203	950	1723	2124	Theoretical Avg
3.747837786	0.24	3.015670342	11.73446328	Percentage Error

Figure 3-2. Set II Simulation Results



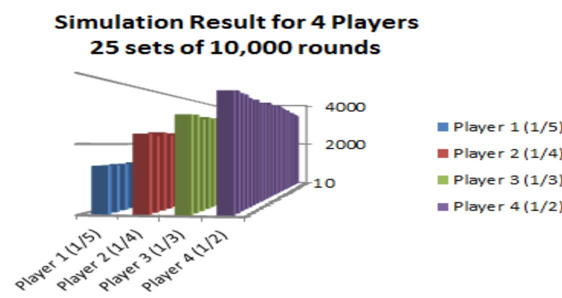
Player 1	Player 2	Player 3	Player 4	Player 5	
4816.2	688	1216.04	1377.48	1902.28	Simulation Avg
4804	666	1195	1361	1832	Theoretical Avg
0.253955	3.303303	1.760669	1.210874	3.836245	Percentage Error

Figure 3-3. Set III Simulation Results



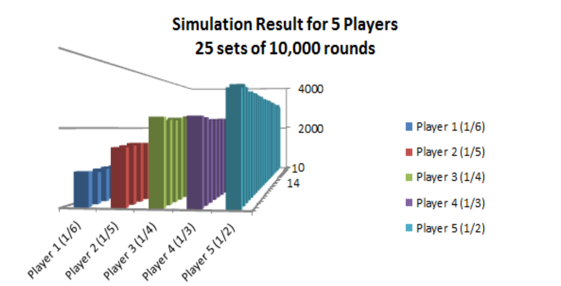
Player 1	Player 2	Player 3	
2336.8	3339.64	4323.56	Simulation Avg
2333	3333	4333	Theoretical Avg
0.162880411	0.199219922	0.217862913	Percentage Error

Figure 3-4. Set IV Simulation Results



Player 1	Player 2	Player 3	Player 4	
1391.64	2319.52	2796.36	3492.48	Simulation Avg
1418	2310	3063	3210	Theoretical Avg
1.858956	0.412121	8.705191	8.8	Percentage Error

Figure 3-5. Set V Simulation Results

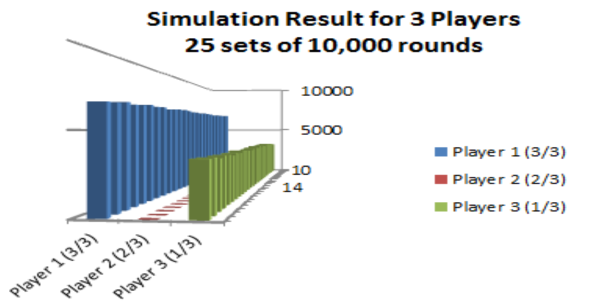


Player 1	Player 2	Player 3	Player 4	Player 5	
881.64	1524.68	2285.4	2257.44	3050.84	Simulation Avg
822	1354	2143	2154	2653	Theoretical Avg
7.255474453	12.605613	6.644890341	4.802228412	14.99585375	Percentage Error

Figure 3-6. Set VI Simulation Results

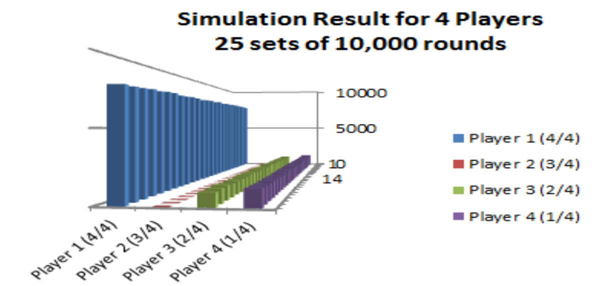
Observations for Sets I-III: Player 1 seems to have the highest chance of survival, followed by player 5, 4, 3, and 2. Simulation results and values derived from MC doesn't seem to be significantly different.

Observations for Sets IV-VI: Player 5 seems to have the highest chance of survival, followed by players 4 and 3 (on tie), then 2 and 1. Simulation results seem to deviate a bit from theoretical values.



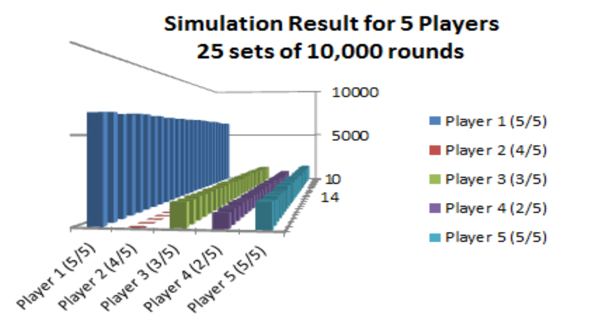
Player 1	Player 2	Player 3	
6676.6	0	3323.4	Simulation Avg
6667	0	3333	Theoretical Avg
0.1439928	0	0.288028803	Percentage Error

Figure 3-7. Set VII Simulation Results



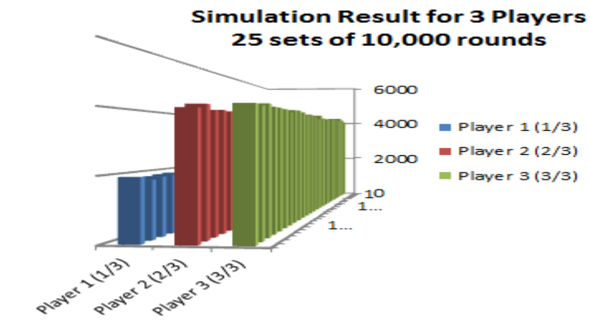
Player 1	Player 2	Player 3	Player 4	
7810.2	0	999.16	1190.64	Simulation Avg
7813	0	1000	1187	Theoretical Avg
0.035838	0	0.084	0.30665543	Percentage Error

Figure 3-8. Set VIII Simulation Results



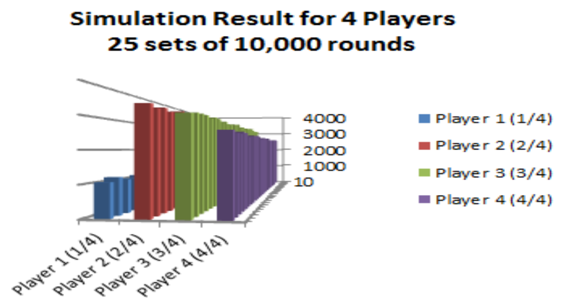
Player 1	Player 2	Player 3	Player 4	Player 5	
6313.04	0	1377.84	855.84	1453.28	Simulation Avg
6305	0	1385	859	1450	Theoretical Avg
0.127518	0	0.516968	0.36787	0.226207	Percentage Error

Figure 3-9. Set IX Simulation Results



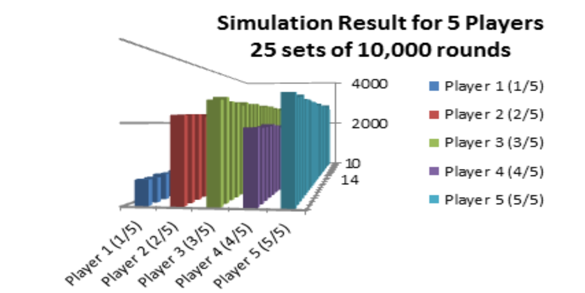
Player 1	Player 2	Player 3	
1907.24	4008.2	4084.56	Simulation Avg
1905	4021	4074	Theoretical Avg
0.1175853	0.31832877	0.2592047	Percentage Error

Figure 3-10. Set X Simulation Results



Player 1	Player 2	Player 3	Player 4	
1091.04	3291.8	3096.36	2520.8	Simulation Avg
1102	3273	3096	2529	Theoretical Avg
0.994555	0.574397	0.011628	0.324239	Percentage Error

Figure 3-11. Set XI Simulation Results



Player 1	Player 2	Player 3	Player 4	Player 5	
649.2	2189.72	2576.76	1891.96	2692.36	Simulation Avg
628	1769	2581	1569	2351	Theoretical Avg
3.375796	23.78293	0.164277	20.58381	14.51978	Percentage Error

Figure 3-12. Set XII Simulation Results

Observations for Sets VII - IX: Player 1 seems to have the highest chance of survival, followed by player 5, 3, 4, and 2. Simulation results and values derived from MC doesn't seem to be significantly different.

Observations for Sets X – XII: This set of result is quite interesting, as a player with perfect shots does not show a dominant chance of survival over other players.

IV. Experimental Method Design

As mentioned earlier in the introduction section, another objective of the research is to compare people's game optimization strategies against the results obtained from Markov Chain long-term transitional probabilities and simulation results. In the experimental method design section, I will discuss the experiment methods and the results.

i. Method

In each experimental session, participants were asked to play four different types of Nuel games. Total of 27 college students at UC Berkeley (a cookie was provided for each participant for incentive☺) were asked to participate in the experiment. For each game type, each participant was asked to choose the player that each participant believed to have the highest chance of survival, with the following rules:

1) Each participant plays 12 sets of the game as illustrated above.
2) Each participant plays each set until there is only one survivor.
3) For every round (i.e. where all players have a chance to shoot exactly once), the participant may change his/her choice of winning player – that is, if the participant believes that a certain group of alive players at given point of time may grant the participant the benefit of changing his/her winning choice, then the participant is welcome to do so.
4) Once a player that the participant chooses is dead, then the participant may not continue to play that set and needs to move onto the next set.

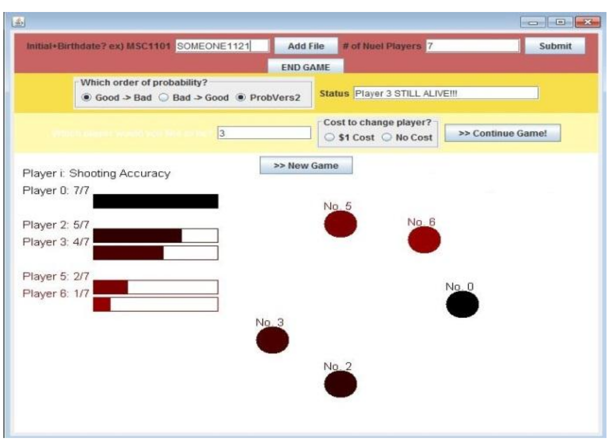


Figure 4. Demonstration of a Nuel experiment program designed on Java Eclipse platform.

The experiment is coded and run on Java platform, as seen on Figure 4. Each circle represents a player, and the level of marksmanship of each player is coded with different colors. Ones with higher level of marksmanship is associated with darker colors, and lower level of marksmanship is associated with brighter colors. Next page illustrates the results from the experiment.

X-axis represents each state (i.e. list of alive players) in Markov Chain, and y-axis represents participant's player choices based on each Markov State.

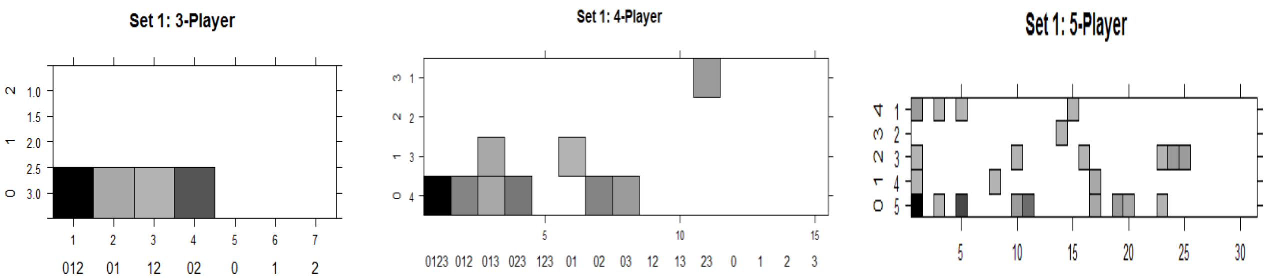


Figure 5-1, 5-2, 5-3: Experiment outcome illustrations for Sets I, II, III

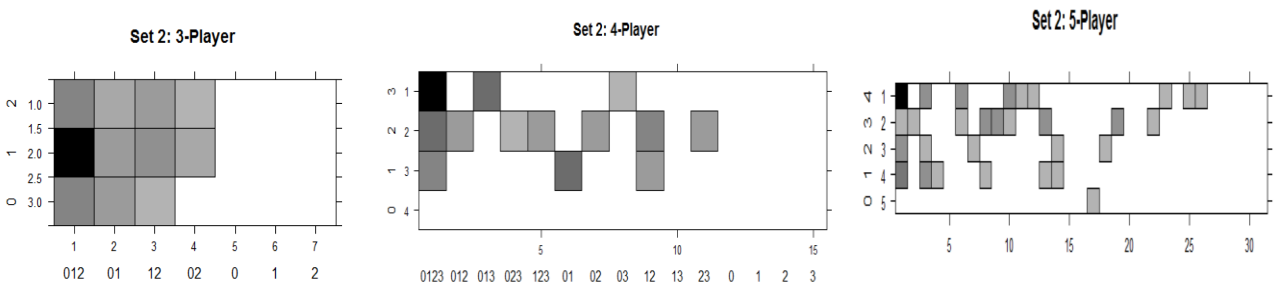


Figure 5-4, 5-5, 5-6: Experiment outcome illustrations for Sets IV, V, VI

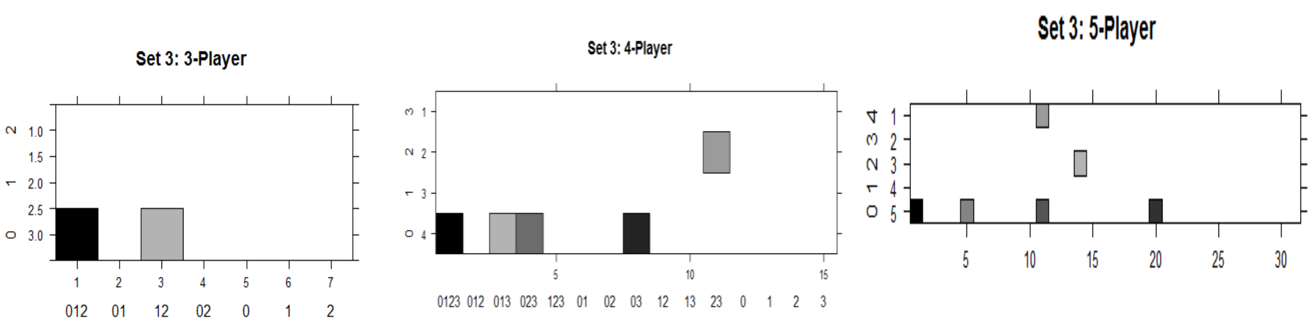


Figure 5-7, 5-8, 5-9: Experiment outcome illustrations for Sets VII, VIII, IX

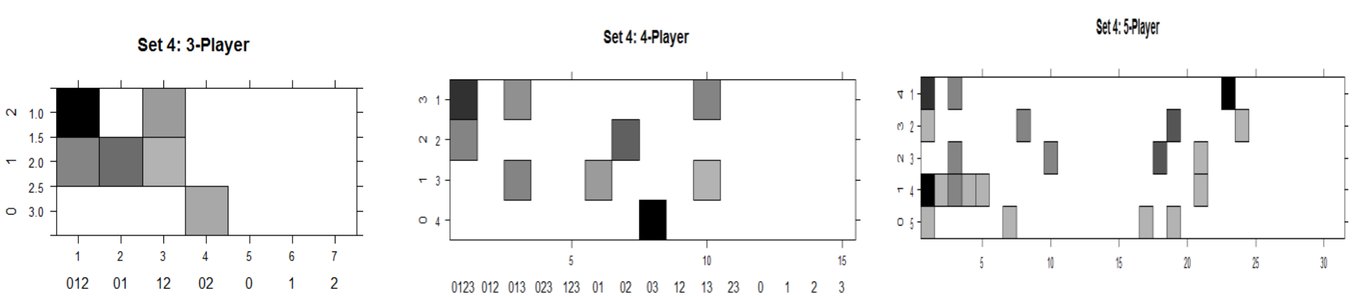


Figure 5-10, 5-11, 5-12: Experiment outcome illustrations for Sets X, XI, XII

V. Conclusion

Theoretical:

As the level of variance of simulation results from theoretical results was shown to be small, turns out that the simulation results match quite closely with the results derived from MC long-transition probabilities. One might be interested in running simulations under different distributions of marksmanship probabilities, such as normal or gamma dist. A further sensitivity analysis is required for further analysis.

Experimental:

I was able to observe variability patterns. Three significant factors that cause variability in participants' choices are: 1) number of players, 2) Presence of a player with perfect shot, 3) Sequence (i.e. choices tend to vary more when weaker players shoot first). While there was no time constraint, I also observed the approximate amount of time each experiment participate spends on making decisions on which player is most likely to win. Experiment results revealed that, the lesser the number of players are out there, the lesser the amount of time the experiment participants are willing to spend, whether or not their choices are optimal strategies. Overall, a significant conclusion is that participants *do* tend to choose players primarily based on accuracy level and not the chance of survival. A further correlation analysis is required to recognize any correlation patterns.

Future Work:

I am currently working on performing sensitivity analysis involving various ranges of player's marksmanship level and number of players involved in the tournament. Matrix decomposition and formulation of linear/non-linear programming is also in progress, if possible. An interesting result may be yielded if one performs extended research on Nuel tournaments involving different situations involving coalitions, which may be a significant contribution to examination of monopolization in business.

VI. References

[1] P. Amengual and R. Toral, "Truels, or Survival of the Weakest," IEEE Transactions Computing in Science & Engineering, September/October 2006, pp.88-89.

Programming Codes Available Upon Request on Github.

Acknowledgement:

I would like to thank Professor Aldous for providing this opportunity to me to freely expand my ideas in this independent project, and always providing encouraging guidance.