Bayes rule: updating probabilities as new information is acquired.

(silly) Example There are 2 coins:
one is fair: $P(\text{Heads}) = 1/2$
one is biased: $P(\text{Heads}) = 9/10$
Pick one coin at random. Toss 3 times. Suppose we get 3 Heads. What then is the chance that the coin we picked is the biased coin?

Abstract set-up: Partition $(B_1, B_2, \ldots)$ of "alternate possibilities".
Know **prior** probabilities $P(B_i)$.
Then observe some event A happens (the "new information") for which we know $P(A|B_i)$. We want to calculate the **posterior** probabilities $P(B_i|A)$.

Bayes formula:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots}.$$

Example used by psychologists in studying how people think about probability.

Two cab companies serve a city: the Green company operates 85% of the cabs and the Blue company operates 15% of the cabs. One of the cabs is involved in a hit-and-run accident, and a witness identifies the hit-and-run cab as a Blue cab. When the court tests the reliability of the witness under circumstances similar to those on the night of the accident, he correctly identifies the colour of a cab 80% of the time and misidentifies it the other 20% of the time. What is the probability that the cab involved in the accident was Blue, as stated by the witness?

Many people answer "80%". What does Bayes formula say?

Example: Suppose that a test for a disease generates the following results:

if a tested patient has the disease, the test returns a positive result 99% of the time.
if a tested patient does not have the disease, the test returns a negative result 95% of the time.
Suppose also that only 0.1% of the population has that disease, so that a randomly selected patient has a 0.001 prior probability of having the disease.

What is the probability that a positive test result is a false positive?

Key point: distinguish between two settings.

This touches upon a **public policy issue**: mass screening for rare diseases has both financial and psychological cost.