Stat 200A Fall 2007 Homework 5

1. Suppose $X_1, \ldots, X_n$ *i.i.d.* $\sim N(\theta, c\theta^2)$, where c is a known constant. Compute the MLE of θ.

2. A company is produces x_i items in the i^{th} week, and Y_i of them turn out to be defective. Here x_i is deterministic and Y_i is random. The engineers think that Y_i is roughly proportional to x_i by some unknown proportionality constant, plus some random error. Formulate a simple two-parameter model for this problem and estimate the parameters (your data is $(x_i, Y_i), i = 1, \ldots, n$).

3. In the above problem, if the engineers think that the fluctuations in Y_i are also proportional to x_i, what is a reasonable model? Compute the MLE's in your model. (Hint: Try $Y_i = \beta x_i + \varepsilon_i$, where ε_i is normal with mean zero and standard deviation proportional to x_i.)

4. Let X be the number of heads in 1000 tosses of a fair coin.

   (a) Using normal approximation via the Central Limit Theorem, find a number k such that $P(500 - k \leq X \leq 500 + k) \simeq 0.95$. You may thus say that $500 \pm k$ is the '95%-range' for the number of heads.

   (b) Suppose in the above experiment the coin is not fair, but has a probability p of landing heads. In terms of p, what is the '95% range' for the number of heads?

   (c) Suppose you don't know p, but the experiment throws up 300 heads in 1000 tosses. If someone tells you that $p = 0.4$, would you be skeptical (based on what you know from part b)?

5. In the above problem, find out all values of p for which 300 lies within the 95% range for the number of heads (i.e., all p for which you would not be 'skeptical') . *This is called the* 95% *confidence interval for the unknown parameter* p.

6. Suppose $X_1, \ldots, X_n$ are i.i.d. $N(\mu, 1)$.

   (a) What is the maximum likelihood estimate of μ?

   (b) What is the 95% confidence interval for the unknown parameter μ if $n = 500$ and you observe that $\overline{X} = 23.4$?