

Calibrating a snow gauge

The data are from a calibration run of the USDA Forest Service's snow gauge located in the Central Sierra Nevada mountain range near Soda Springs, California. The run consists of placing polyethylene blocks of known densities between the two poles of the snow gauge (Figure ??) and taking readings on the blocks. The polyethylene blocks are used to simulate snow.

For each block of polyethylene, 30 measurements were taken. Only the middle 10, in the order taken, are reported here. The measurements recorded by the gauge are an amplified version of the gamma photon count made by the detector. We call the gauge measurements the "gain."

The data available for investigation consist of 10 measurements for each of 9 densities in grams per cubic centimeter (g/cm^3) of polyethylene.

The aim of this project is to provide a simple procedure for converting gain into density when the gauge is in operation. Keep in mind that the experiment was conducted by varying density and measuring the response in gain, but when the gauge is ultimately in use, the snow-pack density is to be estimated from the measured gain.

- Fit gain to density using least squares regression. Compare this fit to one using log of gain. Which fit do the residuals indicate is better?
- We are ultimately interested in answering questions such as: Given a gain reading of 38.6, what is the density of the snow-pack? or Given a gain reading of 426.7, what is the density of the snow-pack? These two numeric values, 38.6 and 426.7, were chosen because they are the average gains for the 0.508 and 0.001 densities, respectively. Add confidence bands around your least squares line that can be used to make interval estimates for the snow-pack density from gain measurements.
- To check how well your procedure works, omit the set of measurements corresponding to the block of density 0.508, apply your calibration procedure to the remaining data, and provide an interval estimate for the density of a block with an average reading of 38.6. Where does the actual density fall in the interval? Try this same test of your procedure for the set of measurements at the 0.001 density.
- Consider the log-polynomial model:

$$\log(\text{gain}) = a + b \times \text{density} + c \times \text{density}^2.$$

How well does it fit the data?

- Use cross-validation to compare these competing models.