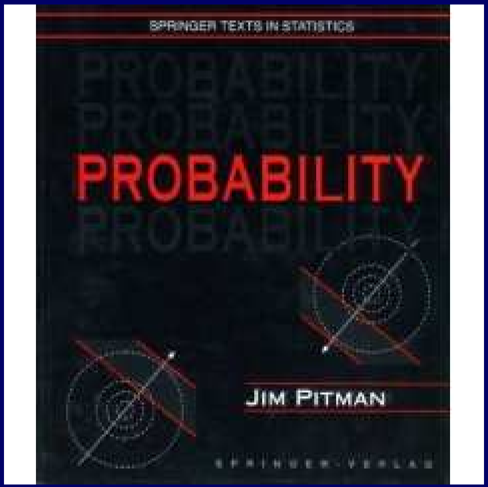
Introduction to probability

Stat 134 FAll 2005

Berkeley



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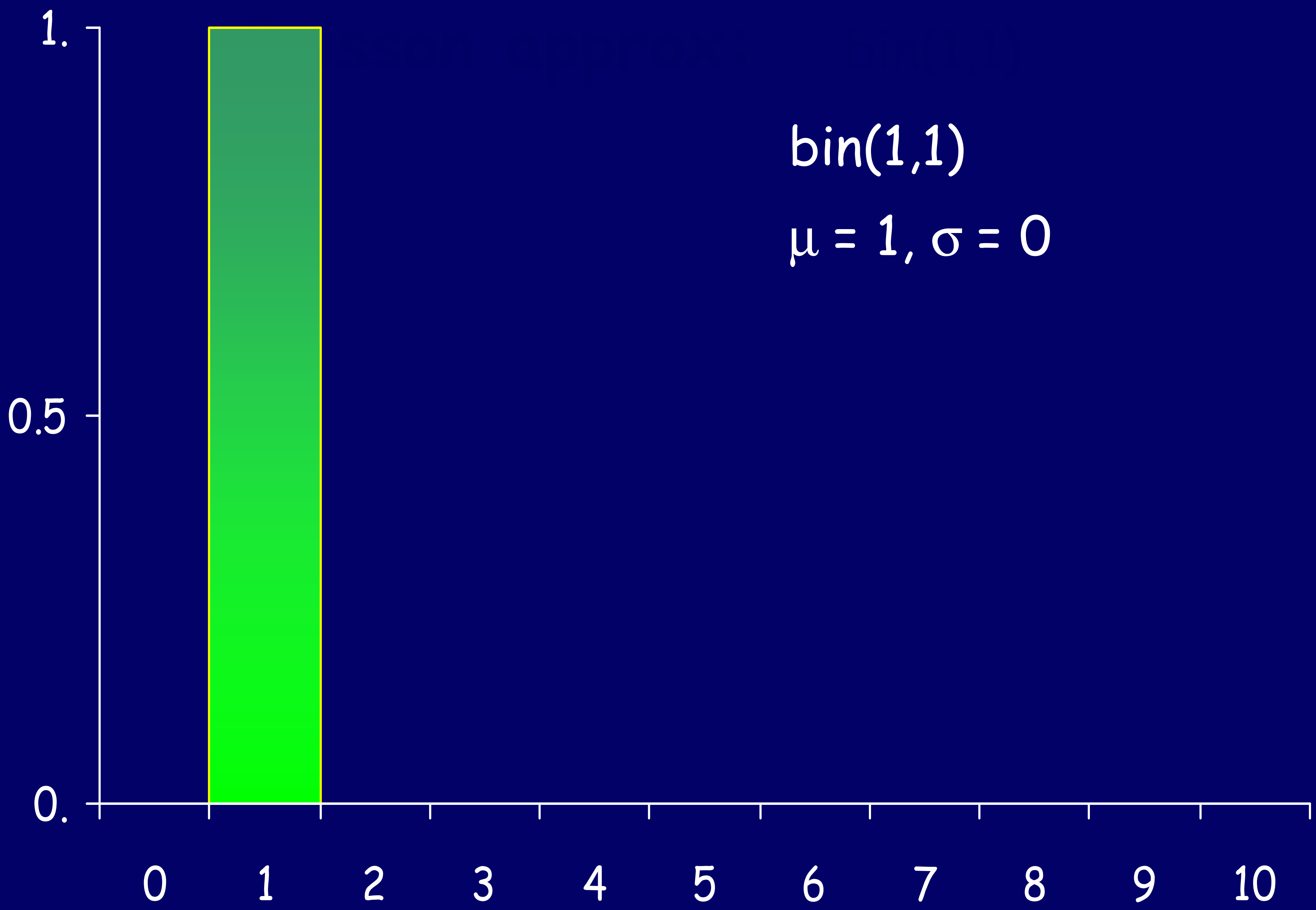
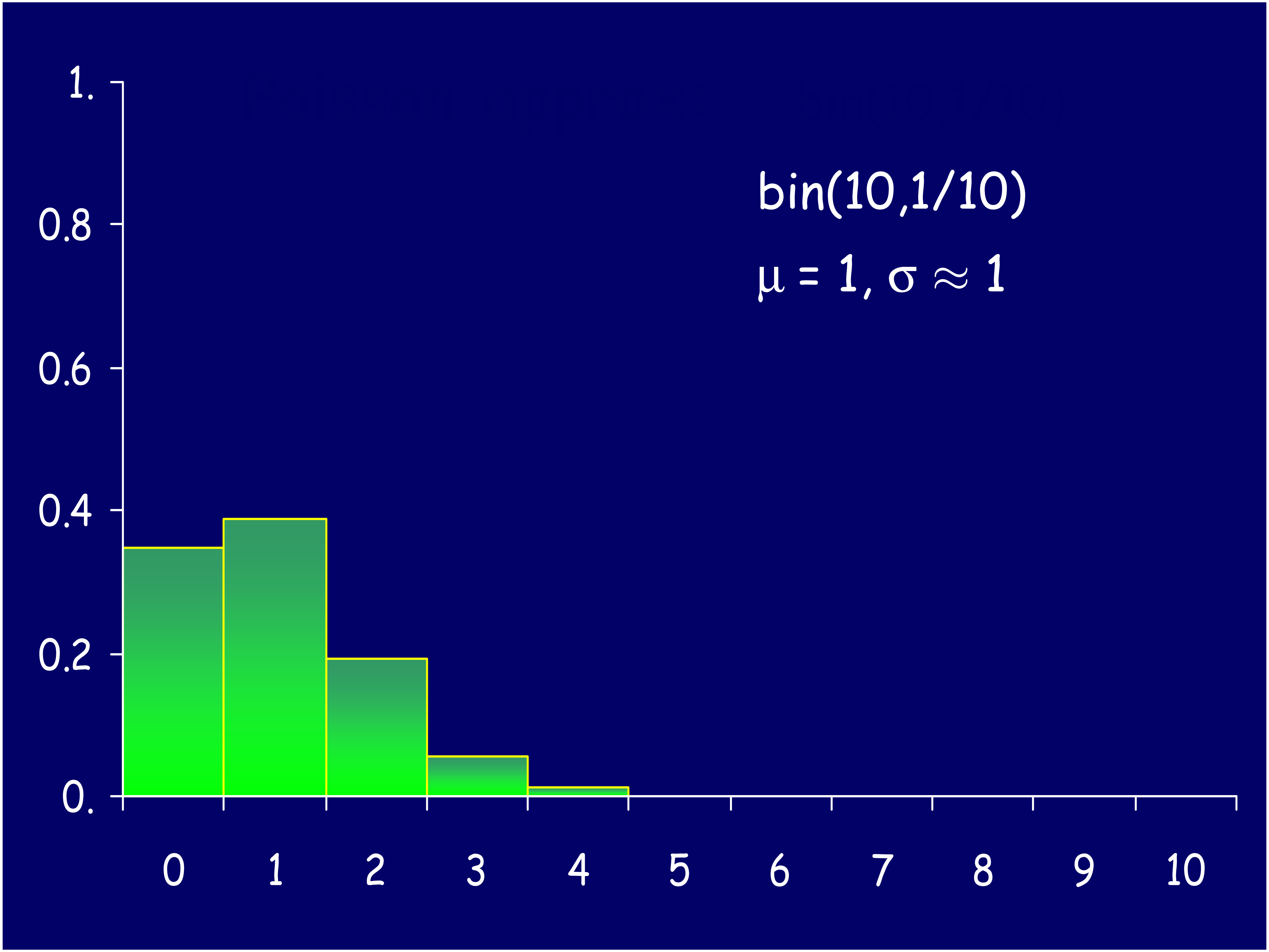
Follows Jim Pitman's book:

Probability

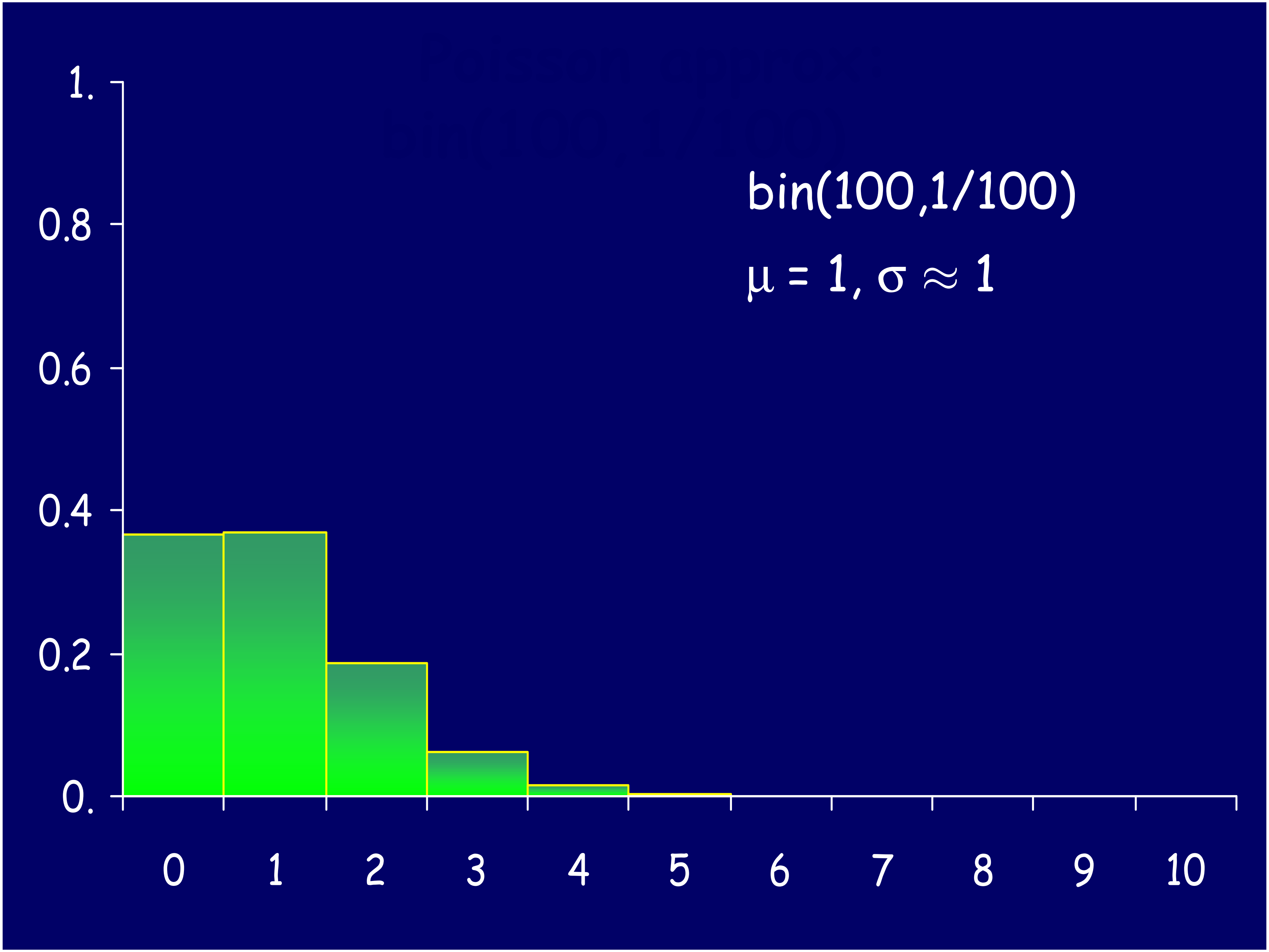
Section 2.4

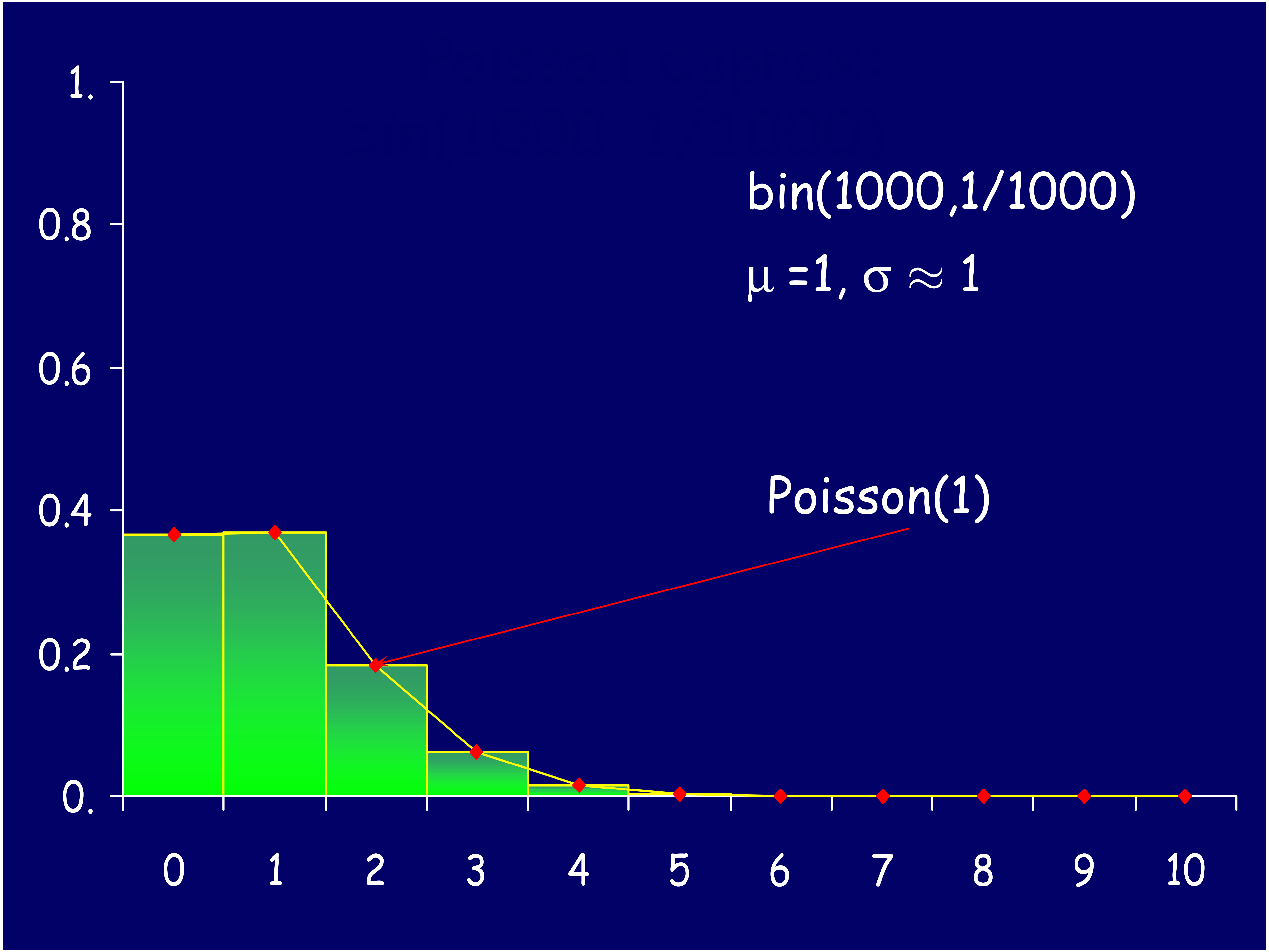


bin(1,1)
μ = 1, σ = 0
1.
0.5
0.
0
1
2
3
4
5
6
7
8
9
10



bin(10,1/10)
μ = 1, σ ≈ 1
1.
0.8
0.6
0.4
0.2
0.
0
1
2
3
4
5
6
7
8
9
10


bin(100,1/100)
μ = 1, σ ≈ 1
1.
0.8
0.6
0.4
0.2
0.
0
1
2
3
4
5
6
7
8
9
10


bin(1000,1/1000)
μ =1, σ ≈ 1
Poisson(1)
1.
0.8
0.6
0.4
0.2
0.
0
1
2
3
4
5
6
7
8
9
10

Example: Summer in Arizona:

100 days of Summer;

Probability of rain 1/100.

$$P(\#RD=0) = (1-1/100)^{100} \approx e^{-1}$$

$$P(\#RD=1) = (100)(1/100)(1-1/100)^{99} \approx e^{-1}$$

$$P(\#RD=2) = (100*99/2)(1/100)^2(1-1/100)^{98}$$

$$\approx \tfrac{1}{2}(99/100)(100/99)^2 e^{-1} \approx \tfrac{1}{2}e^{-1}$$

$$P(\#RD=k) \approx 1/k!\, e^{-1}$$

Poisson Approximation to the Binomial Distribution

If p is small and n is large, the distribution of the number of successes in n independent draws is determined by the mean $\mu = np$, according to the Poisson approximation:

$$P(k \text{ successes}) \approx e^{-\mu} \frac{\mu^k}{k!}$$

The Poisson (μ) Distribution

The Poisson distribution with parameter μ or Poisson(μ) distribution is the distribution of probabilities $P_\mu(k)$ over {0,1,2,...} defined by:

$$P(k) = e^{-\mu} \frac{\mu^k}{k!}$$