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Maximum likelihood solutions for layer parameters based on dynamic surface wave spectra

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Abstract

The paper outlines a probabilistic basis for the automatic estimation from observed surface wave dispersion of structural model parameters with uncertainties. Given an earth model, a wave type and pertinent parameter values, the group velocity of seismic waves can be computed from theory as a function of frequency. Such a function may also be based on the dynamic spectrum representation of an earthquake with known distance and origin time. A likelihood function for the unknown parameters may then be constructed, the parameters estimated and uncertainties assessed. When appropriate, a joint solution may be determined by combining the likelihoods for several stations. The technique is illustrated for Rayleigh waves of the April 20, 1989 Siberian earthquake recorded by a long-period seismograph at the Uppsala, Sweden seismographic station. The purely continental path permits, for the sake of an inversion example, a simple crustal model inversion for three wave modes. © 1997 Elsevier Science B.V.

Keywords: Seismic waves; Earthquakes; Unknown parameters; Rayleigh waves; Siberian earthquake

1. The problem

One of the basic seismological tools to infer Earth structure is the inversion of measurements of the dispersion of surface waves. An example of such a dispersed train for Rayleigh waves is shown in Fig. 1 and Fig. 2. One sees on the vertical component of ground motion the superposition of higher mode waves on the fundamental mode, starting at about 23 min. The method has a long history of application (for historical references, see Aki and Richards, 1980; Bullen and Bolt, 1985), ranging from structural problems of global extent to small-scale problems such as the structure of alluvial basins. Today, regional and global networks of broadband digital seismographs permit the correlation of dispersion properties for surface waves across both two- and three-dimensional structures, such as mountain ranges, grabens and alluvial basins (see, e.g., Mitchell and Herrmann, 1979; Chung and Yeh, 1997; Polet and Kanamori, 1997).

The formal embedding of the problem in a probabilistic framework, however, has not received much treatment to date. Most inversion schemes have used trial-and-error methods (Tarantola, 1987, p. 163). At an early stage of development, graphical comparisons of group or phase velocity curves with measured points were used but, more recently, qualita-

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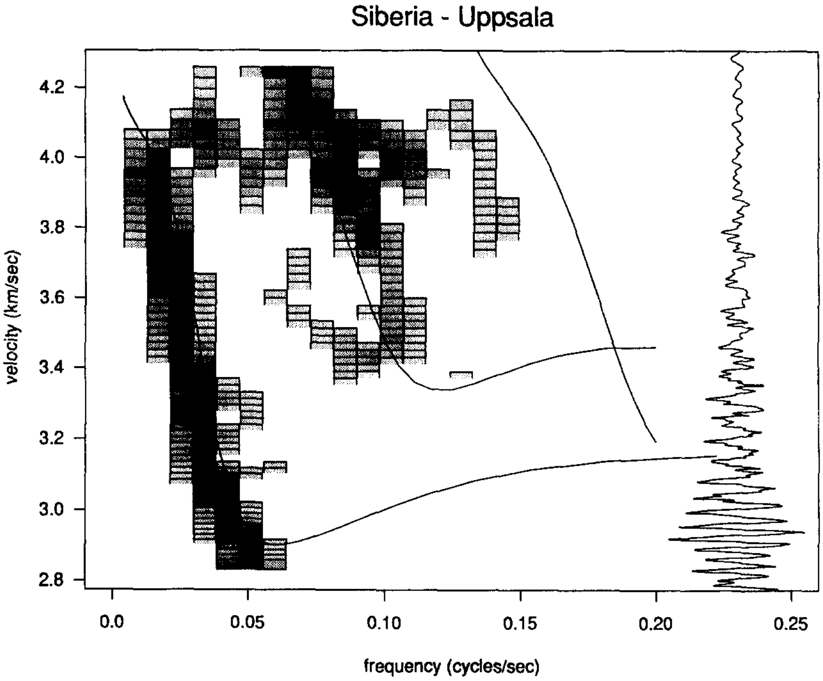

Fig. 1. The dynamic spectrum of the vertical component of a Rayleigh wave train recorded at Uppsala from an earthquake in Siberia on April 20, 1989; epicentral distance, 49°. The vertical curve is the seismogram itself. The three other curves are the estimated dispersion curves of first three Rayleigh modes.

tive algebraic least-squares inversion that minimizes residuals at discrete frequencies in the time domain. The objective of this paper is to present a computational way to apply statistical spectral methods to the estimation of the parameters that define crustal structure, i.e., seismic velocities, density and layer thickness. The goal is to derive spectral estimates based on the surface wave trains in order to make statistically effective inversions to the structural parameters. The approach is one of nonlinear regression analysis that permits assessment of the statistical uncertainties of the computed parameters. It extends directly to allow combination of seismogram measurements from different earthquake sources and stations in relevant inferential problems. It is an advantage that it also works for a single record.

The analysis explored here makes use of the dual nature of the seismic wave field; i.e., a representation either in the frequency domain or the time domain. A dispersed train of travelling surface waves can consequently be analyzed as a superposition of modal vibrations (Bullen and Bolt, 1985, p. 133). Explanation of the observed train of surface waves as a sum of fundamental and higher modes goes back at least to the seismological work of Jeffreys (Jeffreys, 1935).

Since that time, the improvement in band-width of recording seismographs has clearly confirmed that many earthquake sources generate, in layered geological structures, significant vibration modes higher than the fundamental. An example of higher mode Rayleigh wave dispersion has been discussed by MacBeth and Burton (1985). Independent constraints on the inversion schemes require discrimination between these modes (Aki and Richards, 1980, p. 587) and measurement of their spectral properties. Consequently, both discrimination methods and statistical tests must be developed.

2. Theory of the estimation

For seismic Love and Rayleigh waves, the problem to be solved can be formulated as one of nonlinear regression analysis (Seber and Wild, 1989) and dynamic spectral analysis (Brillinger, 1993a). Observational difficulties in wave dispersion inference are well known, arising largely from the changing proportions of Fourier components in the dispersed wave packets as time increases. The presence of significant higher modes in the signal usually diminishes rapidly with modal order, partly due to the nature of typical earthquake sources, but also due to the frequency-dependent damping characteristics of the crust itself. Analysis difficulties include the nonlinearity of the inversion process, and limitations of the data extent and resolution.

Following the discussion of Section 7.1 in Aki and Richards (1980) or Section 3.3.5 in Bullen and Bolt (1985), the seismogram in the case of a single mode may be written as

$$Y(t) = \int \exp(-i(\lambda t - k(\lambda)x))S(\lambda)d\lambda + \text{noise}, \tag{1}$$

where x is the distance from the source, $k(\lambda)$ is the wave number at frequency λ, and $S(\lambda)$ represents the frequency content of the motion at the source.

Set

$$a(t,\lambda) = \left(\frac{1}{x|k''(\lambda)|}\right)^{1/2} \times \exp\left[-i(\lambda t - k(\lambda)x) \pm i\frac{\pi}{4}\right] \tag{2}$$

Then the integral of Eq. (1) may be approximated by

$$S(\lambda_t)\, a(t,\lambda_t), \tag{3}$$

where λ_t is the solution of $xk'(\lambda) = t$, (see Aki and Richards, 1980, Section 7.1 or Bullen and Bolt, 1985, Section 8.7).

In the case that several modes are present, expression (3) is replaced by a sum of terms. This leads one to an approximate representation of the seismogram as

$$Y(t) = \sum_j S(\lambda_{jt})\, a_j(t,\lambda_{jt}) + \text{noise}, \tag{4}$$

where j indexes modes and $xk_j'(\lambda_{jt}) = t$. The function $k(\lambda)$ will depend on θ, the set of unknown parameters (seismic velocities, density and structure lengths) of the Earth models.

The problem is now seen to be one of nonlinear regression analysis and θ and the function $S(.)$ may, for example, be estimated by minimizing the function

$$\sum_t |Y(t) - \sum_j S(\lambda_{jt})\, a_j(t,\lambda_{jt})|^2. \tag{5}$$

Because $S(.)$ is a function, the problem is actually one of semi-parametric analysis (see Bickel et al., 1993). In the results to be presented, $S(.)$ is taken to be piecewise constant on intervals of constant width.

Once estimates of the parameters are available, fitted values and residuals may be obtained by simple substitution and graphed, specifically

$$\hat{Y}(t) = \sum_j \hat{S}(\lambda_{jt})\, \hat{a}_j(t,\lambda_{jt}). \tag{6}$$

In the case that the noise values are independent and identically distributed variates, the estimates minimizing (5) will be maximum likelihood. The estimate of θ may remain efficient even though a high dimensional parameter θ is being estimated (see Bickel et al., 1993). When the noise values are mixing the estimates will be consistent, but generally inefficient. Traditional estimates are available for the standard errors of nonlinear regression estimates (see Seber and Wild, 1989). The uncertainty of the parameters of θ may be estimated by inverting its part of the negative Hessian matrix of the log likelihood function (see Richards, 1961). In the results presented below, this is done assuming the noise values independent and identically distributed normals.

3. Numerical example

The algorithm is illustrated by the use of the Rayleigh wave train shown in Fig. 1. The train is recorded by the vertical component seismograph at Uppsala, and is produced by an earthquake source 49° distant. The earthquake occurred in Siberia on April 20, 1989 with magnitude 6.4. The path of the Rayleigh waves is therefore purely continental. The iteration in this illustration depends on a highly idealized structural model with two crustal layers over a half space that represents the earth's mantle. It is known, of course, that the real continental path involves considerable variation in the thickness of the continental crust, and corresponding variations in the elastic parameters of the rocks. Such complica-

Table 1
Two layers on half space

Initial Values			
Five parameters			
H	α	β	ρ_1/ρ
20.0	5.8	3.45	1.00
20.0	6.5	3.80	1.29
0.0	8.0	4.65	1.48
Estimated values (standard errors in parentheses)			
20.64 (0.54)	5.80	3.45 (0.004)	1.00
19.86 (0.28)	6.50	3.81 (0.02)	1.29
0	8.00	4.66 (0.02)	1.48

tions must be dealt with by multiple recording along the path, in which case the algorithm in this paper can be used for each path segment.

The initial parameter values for the crustal model are given in Table 1; H is the layer thickness in km, α and β are the P and S wave velocities in km/s and ρ is the density. The notation is standard (e.g., Bolt and Butcher, 1960) as are the theoretical computations (Bullen and Bolt, 1985, p. 121). Fig. 1 shows a dynamic spectrum estimate for these data. It suggests the presence of perhaps three modes although the third mode is not well fit at this stage of the iteration. In this work, we consider the case when only the H and β parameters are allowed to vary (dim $\theta = 5$). The function $S(.)$ was taken to have 100 jumps. Fig. 2 provides the fitted signal obtained from Eq. (6) and the residuals. The estimated values for the parameters of the crustal model involved yield a new set of dispersion curves for the fundamental and first two higher modes and these are plotted in Fig. 1. The extent to which the fitted seismogram in Fig. 2 matches some of the minor details of the original motion is impressive. Fig. 3 provides the estimated source power as a function of frequency. This modulates the strength to which the fundamental and higher modes are present in the seismogram. The estimated values and corresponding standard errors of the five parameters are listed in Table 1.

When restricted to just five free parameters, the revised crustal parameters in Table 1 vary in a physically realistic range. The resulting standard errors, calculated as maximum likelihood estimates, were 0.54 and 0.28 km for H_1 and H_2 and formally 0.00, 0.02 and 0.02 for the shear velocities. These values are based on the assumption that the noise values are independent and identically distributed. Fig. 2 suggests the remaining presence of signal

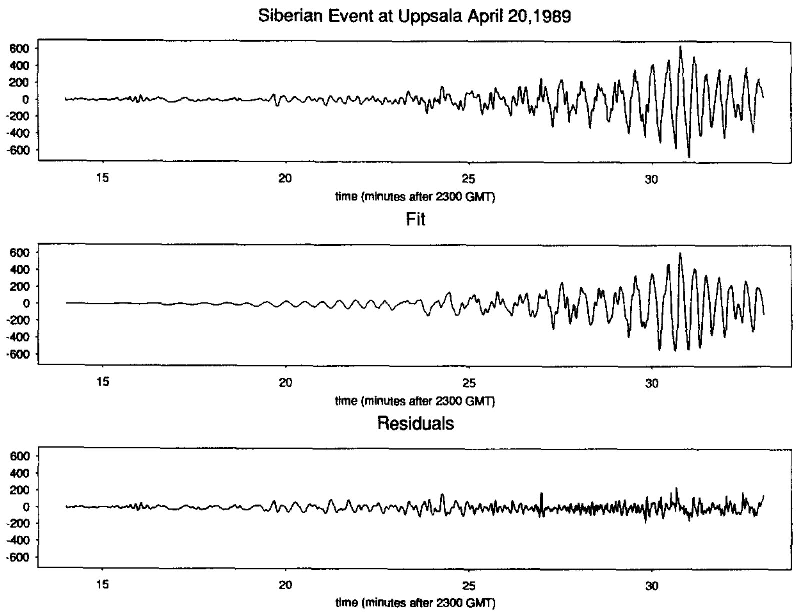

Fig. 2. The top curve is the observed seismogram. The middle is an estimate of the theoretical seismogram as estimated by expression Eq. (6) with three modes. The bottom curve provides the residuals, i.e., the difference between the top and middle curves.

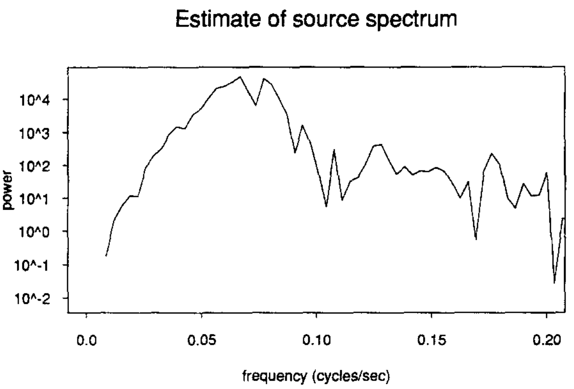

Fig. 3. The estimate of the source spectral power, $|S(\lambda)|^2$, where $S(\lambda)$ is given in Eq. (1).

generated noise as well as seismic wave phases ignored in the analysis. The occurrence of autocorrelation may be anticipated to increase the standard errors. It should be emphasized that the uncertainties in an analysis such as that given here arise not only from measurement error but also from inherent modelling ignorance, such as the effect of multi-pathing, (lateral refraction of the surface waves) and structural variations along the path.

Other initial values were experimented with, but did not change the estimates substantially.

4. Conclusions

The inversion method set out above has a number of advantages typical of semi-parametric modelling in general statistical estimation. It appears to have immediate application as a structural inference tool. First, the maximum likelihood approach can be anticipated to be approximately efficient and this shows in its effectiveness for single seismograms. Secondly, it provides objective measures for comparison with estimates obtained by independent studies (Tarantola, 1987, p. 62). Thirdly, the iterative process illustrated in Section 3 for a single seismogram is easily generalized to the combination of three components of ground motion at one station, or the combination of spectral values for various modal sequences. Joint analysis of group or phase velocity for both Love and Rayleigh wave recordings at a number of stations (e.g., Nolet, 1975; Chung and Yeh, 1997) from different earthquake sources is a straightforward extension. For example, an application of the above dynamic spectral method might allow resolution of difficulties in estimating upper mantle shear velocities from dispersion of both types of long-period surface waves (Polet and Kanamori, 1997). In this case, jackknife estimation of uncertainties may be tried (Brillinger, 1993b), dropping stations and sources in turn.

Finally, the method set out above is presented as an example of the value of statistical display in a complicated geophysical inversion analysis. Firm conclusions on wave modal identification and structural model compatibility can be reached by critical inference from graphical figures. Successive spectral displays with different starting models allow assessment of the appropriateness of the model and provide discrimination of the presence or absence of signal modes. Objective automatic inference along these lines is also a feasible extension of the procedure.

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References

Aki, K., Richards, P.G., 1980. Quantitative seismology. Theory and Methods 2, Freeman, New York.

Bickel, P.J., Klaassen, C.A.J., Ritov, Y., Wallace, J.A., 1993. Efficient and Adaptive Estimation for Semiparametric Models, Johns Hopkins, Baltimore.

Bolt, B.A., Butcher, J., 1960. Rayleigh wave dispersion for a single layer on an elastic half-space. Aust. J. Physics 13, 498–504.

Brillinger, D.R., 1993a. The digital rainbow: some history and applications of numerical spectrum analysis. Can. J. Statistics 21, 1–19.

Brillinger, D.R., 1993b. An application of statistics to seismology: dispersion and modes. In: Rao, T.S. (Ed.), Developments in Time Series Analysis, Chapman & Hall, London, pp. 331–340.

Bullen, K.E., Bolt, B.A., 1985. Introduction to the Theory of Seismology, Cambridge Univ. Press, Cambridge.

Chung, J.-K., Yeh, Y.T., 1997. Shallow crustal structure from short-period Rayleigh-wave dispersion data in southwestern Taiwan. Bull. Seismol. Soc. Am. 87, 370–382.

Jeffreys, H., 1935. The surface waves of earthquakes. Mon. Not. Roy. Astro. Soc., Geophys. Suppl. 3, 253–261.

MacBeth, C.D., Burton, P.W., 1985. Upper crustal shear velocity models from higher mode Rayleigh wave dispersion in Scotland. Geophys. J. R. Astr. Soc. 83, 519–539.

Mitchell, B.J., Herrmann, R.B., 1979. Shear velocity structure in the Eastern United States from the inversion of surface wave group and phase velocities. Bull. Seismol. Soc. Am. 69, 1133–1148.

Nolet, G., 1975. Higher Rayleigh modes in Western Europe. Geophys. Res. Lett. 2, 60–61.

Polet, J., Kanamori, H., 1997. Upper mantle shear velocities beneath Southern California determined from long-period surface waves. Bull. Seismol. Soc. Am. 87, 200–209.

Richards, F.S.G., 1961. A method of maximum likelihood estimation. J. R. Statis. Soc. 23, 469–475.

Seber, G.A.F., Wild, C.J., 1989. Nonlinear Regression, Wiley, New York.

Tarantola, A., 1987. Inverse Problem Theory, Elsevier, Amsterdam.