Anatoly Vershik, Steklov Institute

The infinite dimensional Lebesgue measure, Levy processes with sigma-finite distribution and Poisson-Dirichlet measures

Let $X$ be the unit interval with Lebesgue measure $m$. We define a sigma-finite measure $\mathcal{L}$ on the space of all discrete finite signed measures $\{\sum_k c_k \delta_{x_k}\}$ with $\sum |c_k| < \infty$ (Skorokhod space) which is invariant under the multiplicative abelian group of the functions $a$ with property:

$$\int_X \ln |a(x)| \, dm(x) = 0.$$ 

This measure can be called “infinite dimensional Lebesgue measure”. It is possible to define the one-parametric family of such invariant measures of this type $\mathcal{L}_\theta$ and in a sense these measures can be defined by the “characteristic functional” $\Psi_\theta(f)$:

$$\Psi_\theta(f) = \exp\left\{-\theta \int_X \ln ||f(x)|| \, dx\right\}, \quad \theta > 0.$$ 

The properties of this remarkable measure and its close connections with Levy processes (gamma processes), Poisson-Dirichlet measures $PD(\theta)$ and their generalizations will be discussed.