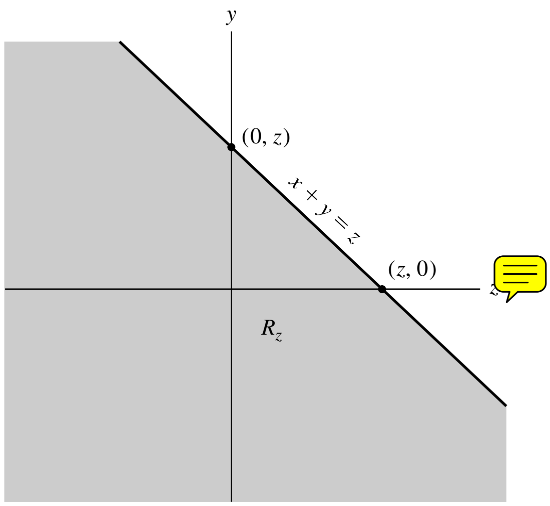

FIGURE **3.17** $X + Y \leq z$ whenever (X, Y) is in the shaded region R_z.

The continuous case is very similar. Supposing that X and Y are continuous random variables, we first find the cdf of Z and then differentiate to find the density. Since $Z \leq z$ whenever the point (X, Y) is in the shaded region R_z shown in Figure 3.17, we have

$$F_Z(z) = \iint_{R_z} f(x, y)\, dx\, dy$$
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{z-x} f(x, y)\, dy\, dx$$

In the inner integral, we make the change of variables $y = v - x$ to obtain

$$F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{z} f(x, v - x)\, dv\, dx$$
$$= \int_{-\infty}^{z} \int_{-\infty}^{\infty} f(x, v - x)\, dx\, dv$$

Differentiating, we have, if $\int_{-\infty}^{\infty} f(x, z - x)\, dx$ is continuous at z,

$$f_Z(z) = \int_{-\infty}^{\infty} f(x, z - x)\, dx$$

which is the obvious analogue of the result for the discrete case.

If X and Y are independent,

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z - x)\, dx$$

This integral is called the **convolution** of the functions f_X and f_Y.