b. Sketch the joint density.
c. Find $P(X^2 + Y^2) \leq \frac{1}{2}$.
d. Find the marginal densities of X and Y. Are X and Y independent random variables?
e. Find the conditional densities.

16. What is the probability density of the time between the arrival of the two packets of Example E in Section 3.4?

17. Let (X, Y) be a random point chosen uniformly on the region $R = \{(x, y) : |x| + |y| \leq 1\}$.
a. Sketch R.
b. Find the marginal densities of X and Y using your sketch. Be careful of the range of integration.
c. Find the conditional density of Y given X.

18. Let X and Y have the joint density function

$$f(x, y) = k(x - y), \qquad 0 \leq y \leq x \leq 1$$

and 0 elsewhere.
a. Sketch the region over which the density is positive and use it in determining limits of integration to answer the following questions.
b. Find k.
c. Find the marginal densities of X and Y.
d. Find the conditional densities of Y given X and X given Y.

19. Suppose that two components have independent exponentially distributed lifetimes, T_1 and T_2, with parameters α and β, respectively. Find (a) $P(T_1 > T_2)$ and (b) $P(T_1 > 2T_2)$.

20. If X_1 is uniform on $[0, 1]$, and, conditional on X_1, X_2, is uniform on $[0, X_1]$, find the joint and marginal distributions of X_1 and X_2.

21. An instrument is used to measure very small concentrations, X, of a certain chemical in soil samples. Suppose that the values of X in those soils in which the chemical is present is modeled as a random variable with density function $f(x)$. The assay of a soil reports a concentration only if the chemical is first determined to be present. At very low concentrations, however, the chemical may fail to be detected even if it is present. This phenomenon is modeled by assuming that if the concentration is x, the chemical is detected with probability $R(x)$. Let Y denote the concentration of a chemical in a soil in which it has been determined to be present. Show that the density function of Y is

$$g(y) = \frac{R(y)f(y)}{\int_0^\infty R(x)f(x)\,dx}$$

22. Consider a Poisson process on the real line, and denote by $N(t_1, t_2)$ the number of events in the interval (t_1, t_2). If $t_0 < t_1 < t_2$, find the conditional distribution of $N(t_0, t_1)$ given that $N(t_0, t_2) = n$. (*Hint:* Use the fact that the numbers of events in disjoint subsets are independent.)