Close enough for government work: Risk-Limiting Post-Election Audits

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Outline

Abstract

How can we tell whether votes were tallied accurately enough to determine the correct winners of an election? Risk-limiting post election audits, endorsed by the American Statistical Association, Common Cause, The League of Women Voters, Verified Voting, and other groups concerned with election integrity, can help. A *risk-limiting audit* has a guaranteed minimum chance of correcting electoral errors. There have been seven risk-limiting audits so far, six in California and one in Colorado. California AB 2023, which requires an official pilot of risk-limiting audits, was signed into law in July, 2010 after unanimous, bipartisan votes in both legislative bodies. I will discuss the theory behind risk-limiting audits (couching auditing as a sequential nonparametric hypothesis test about the mean of a bounded population); factors that affect efficiency and cost; "transitive auditing," which uses a shadow system rather than the system of record; and lessons learned conducting audits and working with elections officials and election integrity activists. If time permits, I'll ramble about getting AB 2023 endorsed and passed and about the recent tangle in New York Senate District 7, which shifted the balance of power in the NY senate: They are interesting case studies at the intersection of statistics, policy, legislation, public administration, jurisprudence, and politics.

[Election Leak] [CNN: DC hack] [Voting Machine Wins] [Homer Votes—sort of]

Vote-flipping in North Carolina

NC GOP leader: Touchscreen voting machines have programming flaw, by Michael Biesecker

The chairman of the N.C. Republican Party alleged Thursday that a programming flaw with touchscreen voting machines used for early voting in 36 counties is causing votes intended for GOP candidates to be counted for Democrats.

Tom Fetzer, the Republican chairman, said that if the State Board of Elections does not enact a list of demands intended to remedy the problem by the end of today, the party's lawyers will be in federal court Friday morning seeking a statewide injunction.

. . .

Johnnie McLean, deputy director of the state elections board, said Thursday that her office has received no widespread reports of problems.

"In every election we will have scattered reports of machines where the screens need to be recalibrated," McLean said. "That sort of comes with the territory with touch-screen technology."

NEWS OBSERVER, 28 OCTOBER 2010, `http://www.newsobserver.com/2010/10/28/766257/nc-republican-party-chair-touchscreen.html#ixzz13gTJCCvp`

Humboldt County CA, 2008

Serious Error in Diebold Voting Software Caused Lost Ballots in California County, by Kim Zetter

Election officials in a small county in California discovered by chance last week that the tabulation software they used to tally votes in this year's general election dropped 197 paper ballots from the totals at one precinct. The system's audit log also appears to have deleted any sign that the ballots had ever been recorded.

Premier has acknowledged . . . its software caused the system to delete votes. The company has apparently known about the problem since 2004 . . .

[RoV] Crnich would never have discovered the problem through her standard canvassing procedures . . . nor would she have discovered it while conducting a mandatory manual audit that California counties are required to do.

Crnich discovered the missing ballots only because she happened to implement a new and innovative auditing system this year that was spearheaded by members of the public who helped her develop it.

WIRED NEWS, 8 DECEMBER 2008, `http://blog.wired.com/27bstroke6/2008/12/unique-election.html`

Polk County NC, 2008

Owens victory in Polk is in doubt, by Times-News staff

Ted Owens went to sleep Tuesday night thinking he had earned another term . . . A recount Wednesday showed he may not have. . . .

Computer software initially displayed figures that were different than those shown by the voting machines . . .

The software installed in the stand-alone computer that ballot results are fed into was the problem . . . [Elections Director Dale Edwards] said there was no explanation as to why the computer counted the wrong numbers, and no one is at fault.

BLUERIDGENOW.COM TIMES-NEWS, 6 NOVEMBER 2008, `http://www.blueridgenow.com/article/20081106/NEWS/811050255`

Santa Clara County, CA, 2008

Few problems reported in area despite record turnout, by Karen de Sá and Lisa Fernandez

Record-high voting in the Bay Area on Tuesday mostly defied predictions of unwieldy waits and overwhelmed polls. But in Santa Clara County, concerns about touch-screen voting machines will likely increase following significant malfunctions.

Fifty-seven of the county's Sequoia Voting Systems machines failed on Election Day, resulting in hourslong delays before replacements arrived.

MERCURY NEWS, 4 NOVEMBER 2008, `http://www.mercurynews.com/elections/ci_10901166?nclick_check=1`

Leon County, FL, 2008

Ballots not being recorded at two Leon County polling places, by Angeline J. Taylor

Leon County Supervisor of Elections Ion Sancho has reported that ballots . . . are not being read properly. The problem, he said, rests with a new machine that has been purchased for polling sites throughout the state. . . .

"Certain ballots are being rejected across the state," he said. . . . If the machine reads the ballot card as too long, the . . . machine will simply not read the card.

TALLAHASSEE DEMOCRAT, 20 OCTOBER 2008, `http://www.tallahassee.com/article/20081020/BREAKINGNEWS/81020024`

Palm Beach County, FL, 2008

Florida Primary Recount Surfaces Grave Voting Problems One Month Before Presidential Election, by Kim Zetter

At issue is an August 26 primary election in which officials discovered, during a recount of a close judicial race, that more than 3,400 ballots had mysteriously disappeared after they were initially counted on election day. The recount a week later, minus the missing ballots, flipped the results of the race to a different winner.

. . . officials found an additional 227 ballots that were never counted on election day . . . in boxes in the county's tabulation center.

Palm Beach County was using new optical-scan machines that it recently purchased from Sequoia Voting Systems for $5.5 million.

Palm Beach County, FL, 2008, cont'd

[In a re-scan of ballots the machines had rejected] [o]fficials expected the machines would reject the same ballots again. But that didn't happen. During a first test of 160 ballots, the machines accepted three of them. In a second test of 102 ballots, the machines accepted 13 of them . . . When the same ballots were run through the machines again, 90 of the ballots were accepted.

[T]he county then re-scanned two batches of 51 ballots each that had initially been rejected for having no vote cast in the judicial race, but that were found in a manual examination to contain legitimate votes for one candidate or the other. The first batch of 51 ballots were found to have legitimate votes for Abramson. The second batch of 51 ballots were found to have legitimate votes for Wennet.

In the first batch of 51 ballots . . . 11 of the ballots that had previously been rejected as undervotes were now accepted . . . the remaining 40 ballots were rejected as having no votes. In the second batch of 51 ballots . . . the same machine accepted 2 ballots and rejected 49.

Palm Beach County, FL, 2008, cont'd

The same two batches of ballots were then run through the second . . . machine. [I]n the first batch . . . the machine accepted 41 . . . and rejected 10 others. In the second batch . . . the machine accepted 49 of the ballots and rejected 2—the exact opposite of the results from the first machine.

WIRED NEWS, 7 OCTOBER 2008, `http://blog.wired.com/27bstroke6/2008/10/florida-countys.html`

Washington, DC, 2008

Report Blames Speed In Primary Vote Error; Exact Cause of Defect Not Pinpointed, by Nikita Stewart

Speed might have contributed to the Sept. 9 primary debacle involving thousands of phantom votes, according to a D.C. Board of Elections and Ethics report issued yesterday. . . . [T]he report does not offer a definitive explanation. . .

The infamous Precinct 141 cartridge "had inexplicably added randomly generated numbers to the totals that had been reported," according to the report written by the elections board's internal investigative team.

. . . 4,759 votes were reflected instead of the actual 326 cast there.

WASHINGTON POST, 2 OCTOBER 2008; PAGE B02

see also hearings at
`http://www.octt.dc.gov/services/on_demand_video/channel13/October2008/10_03_08_PUBSVRC_2.asx`

New Jersey 2008

County finds vote errors: Discrepancies discovered in 5% of machines, by Robert Stern

Five percent of the 600 electronic voting machines used in Mercer County during the Feb. 5 presidential primary recorded inaccurate voter turnout totals, county officials said yesterday . . .

23 FEBRUARY 2008, NEW JERSEY TIMES

Ohio 2004

Machine Error Gives Bush Thousands of Extra Ohio Votes, by John McCarthy

COLUMBUS, Ohio – An error with an electronic voting system gave President Bush 3,893 extra votes in suburban Columbus, elections officials said. Franklin County's unofficial results had Bush receiving 4,258 votes to Democrat John Kerry's 260 votes in a precinct in Gahanna. Records show only 638 voters cast ballots in that precinct. Bush's total should have been recorded as 365.

5 NOVEMBER 2004, ASSOCIATED PRESS

Florida 2004

Broward Machines Count Backward, by Eliot Kleinberg

[E]arly Thursday, as Broward County elections officials wrapped up after a long day of canvassing votes, something unusual caught their eye. Tallies should go up as more votes are counted. Thats simple math. But in some races, the numbers had gone . . . down.

Officials found the software used in Broward can handle only 32,000 votes per precinct. After that, the system starts counting backward.

. . . The problem cropped up in the 2002 election. . . . Broward elections officials said they had thought the problem was fixed.

5 NOVEMBER 2004, THE PALM BEACH POST

What's the issue?

- Any way of counting votes makes mistakes.
- If there are enough mistakes, apparent winner could be wrong.
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What's important here?

Crucial question:

When to *stop* auditing, not how many ballots to audit initially.

Solution:

If there's compelling evidence that outcome is right, stop; else, audit more.

Current audit laws have the wrong focus: essentially useless for correcting wrong outcomes. (California just passed AB 2023, which calls for a pilot of a statistically sound approach: risk-limiting audits.)

Efficiency is primarily about batch sizes: Need data plumbing.

California Elections Code §15360

[T]he official conducting the election shall conduct a public manual tally of the ballots tabulated by those devices, including absent voters' ballots, cast in 1 percent of the precincts chosen at random by the elections official . . .

The elections official shall use either a random number generator or other method specified in regulations . . .

The official conducting the election shall include a report on the results of the 1 percent manual tally in the certification of the official canvass of the vote. This report shall identify any discrepancies between the machine count and the manual tally and a description of how each of these discrepancies was resolved . . .

NJ S507 [1R] (Gill)

[officials] shall conduct random hand counts of the voter-verified paper records in at least two percent of the election districts where elections are held for federal or State office . . .

Any procedure designed, adopted, and implemented by the audit team shall be implemented to ensure with at least 99% statistical power that for each federal, gubernatorial or other Statewide election held in the State, a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit . . .

[procedures] shall be based upon scientifically reasonable assumptions . . . including but not limited to: the possibility that within any election district up to 20% of the total votes cast may have been counted for a candidate or ballot position other than the one intended by the voters[.]

Say what?

Others

Oregon and New Mexico have audit laws that allow the sample (of races and/or ballots) to be selected before the election.

Rep. Rush Holt has proposed federal legislation that has tiered sampling fractions, depending on the margin—but no requirement for followup if errors are found.

Can't correct wrong outcomes without counting the whole audit trail.

What should an election audit law do?

Legislation should enunciate *principles*, not *methods*.

Methods are best left to regulation: Easier to improve, fix, etc.

Mutual distrust among election integrity advocates, elections officials, and legislators is an unfortunate but important consideration.

California AB 2023 (Saldaña, sponsored by SoS Bowen)

First proposed audit bill that limits risk!

(b)(3) “Risk-limiting audit” means a manual tally employing a statistical method that ensures a large, predetermined minimum chance of requiring a full manual tally whenever a full manual tally would show an electoral outcome that differs from the outcome reported by the vote tabulating device for the audited contest. A risk-limiting audit shall begin with a hand tally of the votes in one or more audit units and shall continue to hand tally votes in additional audit units until there is strong statistical evidence that the electoral outcome is correct. In the event that counting additional audit units does not provide strong statistical evidence that the electoral outcome is correct, the audit shall continue until there has been a full manual tally to determine the correct electoral outcome of the audited contest.

`http://www.leginfo.ca.gov/pub/09-10/bill/asm/ab_2001-2050/ab_2023_bill_20100325_amended_asm_v98.html`

Role of statistics

Limiting the risk is easy

No statistics needed: just count all the ballots by hand.

Statistics lets you do less counting when the outcome is right, but still ensure a big chance of a full hand count when outcome is wrong.

Definitions

Outcome: set of winners, not exact vote totals.

Machine-count outcome, apparent outcome: outcome that will become officially final unless an audit or other action intervenes.

Apparent winner: won according to apparent outcome

Hand-count outcome, true outcome, correct outcome: outcome a full manual tally of the audit trail would show

True winner: would win according to full hand tally, if there were a full hand tally

more definitions . . .

Risk-limiting audit: audit with guaranteed minimum chance of correcting a wrong outcome (by counting the whole audit trail). Endorsed by ASA, CC, VV, LWV, CEIMN, . . .

Risk: maximum chance that the audit fails to correct an apparent outcome that is incorrect, no matter what caused the outcome to be incorrect.

Simultaneous risk-limiting audit: guaranteed minimum chance of correcting *all* the contests that have incorrect apparent outcomes.

Simultaneous risk: the maximum chance that the audit won't correct one or more of the apparent outcomes that are incorrect

Crucial ingredients for risk-limiting audits

- Essential that voters create complete, durable, accurate audit trail.
- Essential that voting systems enable auditors to access reported results (total ballots, counts for each candidate, registered voters) in auditable batches. (Smaller batches are better.)
- Essential to select batches at random, *after* the results are posted. (Can supplement with "targeted" samples.)
- Need a plan for dealing with discrepancies, possibly leading to full count. "Explaining" or "resolving" isn't enough.
- Only one current audit law limits risk: AB 2023. CO HB 09-1335 requires "risk-limiting audits" but doesn't define "risk."
- Compliance audits vs. materiality audits.

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Assessing Evidence

- How strong is the evidence that the outcome is correct, given how the sample was drawn, the margin, the errors found, etc.?
- What is the biggest chance that—if the outcome is wrong—the audit would have found as little error as it did? (Define "little" for tractability and power.)
- (Maximum) *P*-value of the hypothesis that the apparent outcome of one or more contests is wrong.

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MACRO

Sufficient condition for all outcomes to be right:

For every winner and loser, the overstatement errors minus the understatement errors amount to less than 100% of the margin between that pair of candidates.

MACRO (maximum across-race relative overstatement) summarizes overstatement errors within and across contests.

If the MACRO summed over all ballots is less than 100%, all outcomes of all contests are correct.

MACRO

For $w \in \mathcal{W}_c$, $\ell \in \mathcal{L}_c$, define

$$e_{pw\ell} \equiv \begin{cases} \frac{(v_{wp} - v_{\ell p}) - (a_{wp} - a_{\ell p})}{V_{w\ell}}, & \text{if batch } p \text{ contains contest } c \\ 0, & \text{otherwise.} \end{cases}$$

If any apparent outcome is wrong,

$$\exists c \in \{1, \ldots, C\} \text{ s.t. } \exists (w \in \mathcal{W}_c,\ \ell \in \mathcal{L}_c) \text{ with } \sum_{p=1}^{N} e_{pw\ell} \geq 1. \tag{1}$$

Test based on sufficient condition

$$e_p \equiv \max_c \max_{w \in \mathcal{W}_c,\, \ell \in \mathcal{L}_c} e_{pw\ell}.$$

All outcomes must be correct if

$$E \equiv \sum_{p=1}^{N} e_p < 1.$$

Maximum across-contest relative overstatement of margins (MACRO)

Controlling the familywise error rate

C null hypotheses,

$$\textit{the outcome of contest } c \textit{ is incorrect, } c = 1, \ldots, C.$$

If $E < 1$, the entire family of C null hypotheses is false: all apparent outcomes are right.

Test of hypothesis $E \geq 1$ at significance level α is a test of the C hypotheses with familywise error rate no larger than α.

Bounding the overstatement error in each batch

A priori bounds are crucial.

If number of valid ballots cast in batch p for contest c is at most b_{cp} then

$$e_{pw\ell} \leq (v_{wp} - v_{\ell p} + b_{cp})/V_{w\ell}.$$

Hence,

$$e_p \leq \max_{c \in \{1,\ldots,C\}} \max_{w \in \mathcal{W}_c, \ell \in \mathcal{L}_c} \frac{v_{wp} - v_{\ell p} + b_{cp}}{V_{w\ell}} \equiv u_p.$$

u_p is a limit on e_p, the maximum relative overstatement of *any* margin that can be concealed in batch p, the MACRO in batch p.

$U \equiv \sum_p u_p$, upper bound on MACRO.

Sampling Designs

- For most voting technology, limited to some kind of cluster sample.
- Simple, Stratified (by county, voting method, other), PPEB/PPS, NEGEXP, Stratified PPEB?
- Sampling scheme affects choice of test statistic—analytic tractability
- Weighted max, binning for simple & stratified sampling, NEGEXP, PPEB.
- More efficient choices possible for PPEB: Kaplan-Markov, Feige?

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Taint

taint of batch p

$$\tau_p = \frac{e_p}{u_p} \leq 1.$$

Draw batches with replacement. In each draw,

$$\mathbf{P}\{\text{draw batch } p\} = u_p/U.$$

PPS, used in financial auditing.

Taint of jth draw is T_j. $\{T_j\}$ are iid, $\mathbf{E}T_j = E/U$.

Can stop the audit if can reject the hypothesis $\mathbf{E}T_j \geq 1/U$.

Reduces auditing to testing hypothesis about the mean of a bounded random variable.

Sequential risk-limiting audit using Kaplan-Markov bound

0. Calculate error bounds $\{u_p\}$, U. Set $n = 1$. Pick $\alpha \in (0, 1)$ and $m > 0$.
1. Draw a batch using PPEB. Audit it if it has not already been audited.
2. Find $T_n \equiv t_p \equiv e_p/u_p$, taint of the batch p drawn at stage n.
3. Compute

$$P_n \equiv \prod_{j=1}^{n} \frac{1 - 1/U}{1 - T_j}. \quad \text{See November 2010 WIRED, p.56} \quad (2)$$

- 4. If $P_n < \alpha$, report apparent outcomes and stop. If $n = m$, audit remaining batches, report then-known outcomes and stop.. Else, $n \leftarrow n + 1$ and go to 1.

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This sequential procedure is risk-limiting

Chance $\geq 1 - \alpha$ of correcting wrong outcomes by full hand count

If any outcome is wrong,

$$\mathbf{P}\{\text{stop without auditing every batch}\} < \alpha.$$

Remarkably efficient if batches are not too big.

Feige's Inequality—sharper than Kaplan-Markov?

He, Zhang, and Zhang, 2010 (Theorem 3.2)

$\{X_j\}_{j=1}^n$ independent; $\mathbf{E}X_j = 0$, $\forall j$. Fix $\Delta > 0$. Suppose $\exists c > 0$ s.t. $X_j \geq -c\Delta$ $\forall j$. Let $S \equiv \sum_{j=1}^n X_j$. Then for any $\tau > 0$,

$$\mathbf{P}\{S < \Delta\} \geq e^{-1/\tau} F(c, \tau \max(1, c)), \tag{3}$$

where

$$F(c_1, c_2) \equiv (2\sqrt{3} - 3) \frac{4(s(c_1, c_2) + 2)}{s^2(c_1, c_2) + 12 s(c_1, c_2) + 24} \tag{4}$$

and

$$s(c_1, c_2) \equiv \max\{c_1^2 + 4c_1, c_2^2 - 4c_2, c_1^2 + c_2^2 - 4c_1 c_2 - 4(c_2 - c_1)\} \tag{5}$$

Pilot Audits in California

Marin County (February 2008; November 2008, 2009)

Yolo County (November 2008, 2009)

Santa Cruz County (November 2008)

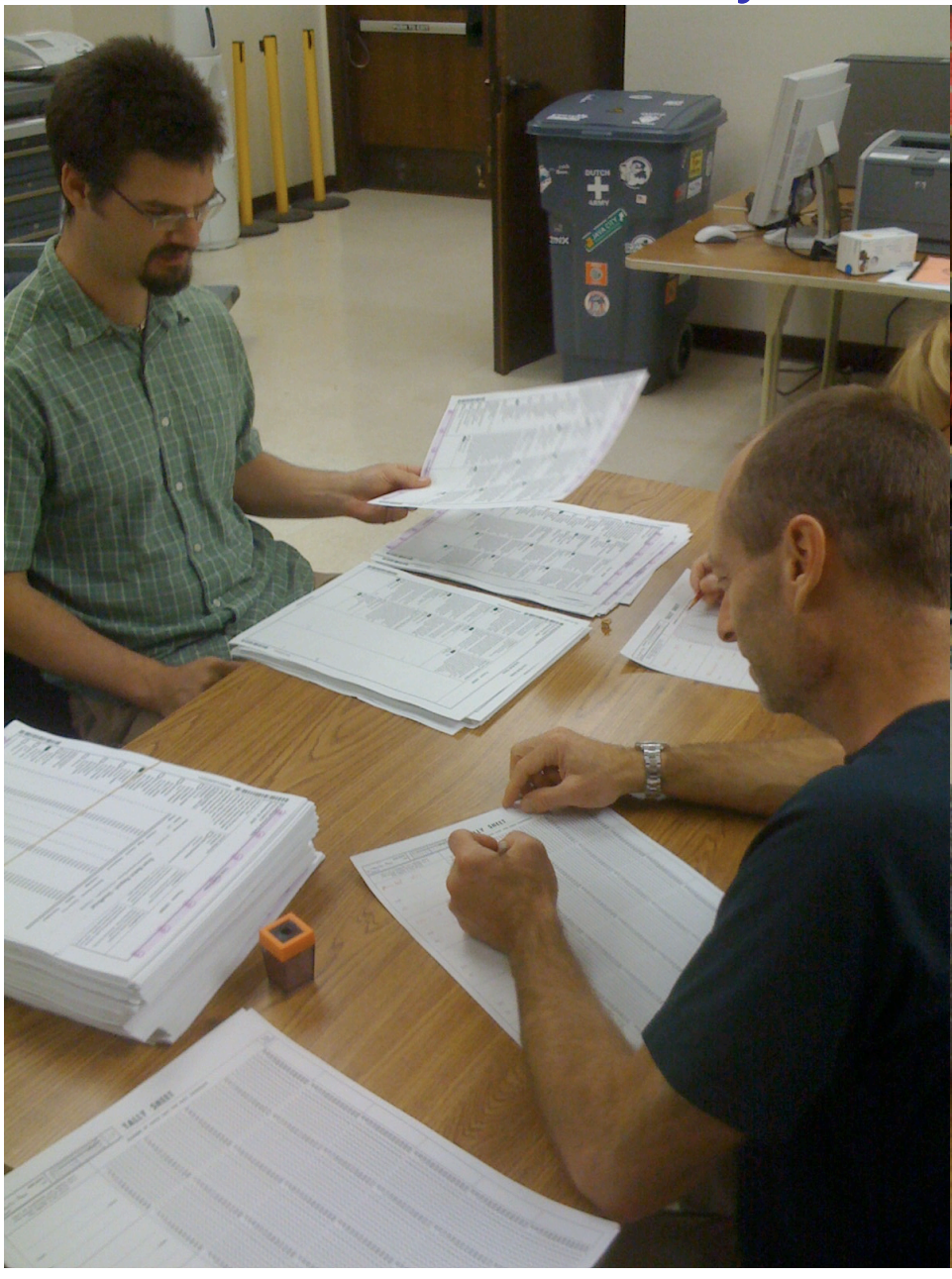
Measures requiring super-majority, simple measures, multi-candidate contests, vote-for-*n* contests.

Contest sizes ranged from about 200 ballots to 121,000 ballots.

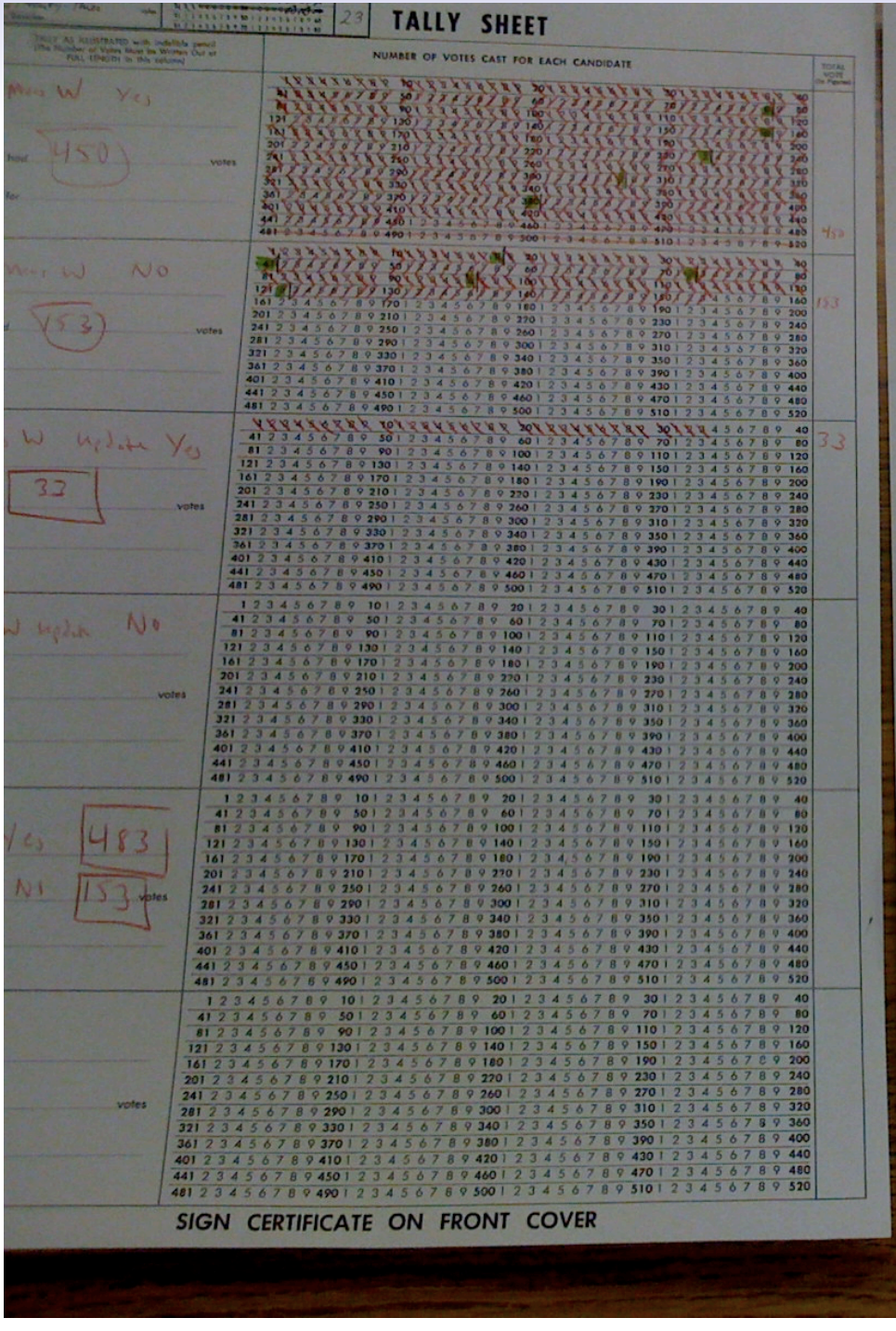
Counting burden ranged from 32 ballots to 7,000 ballots.

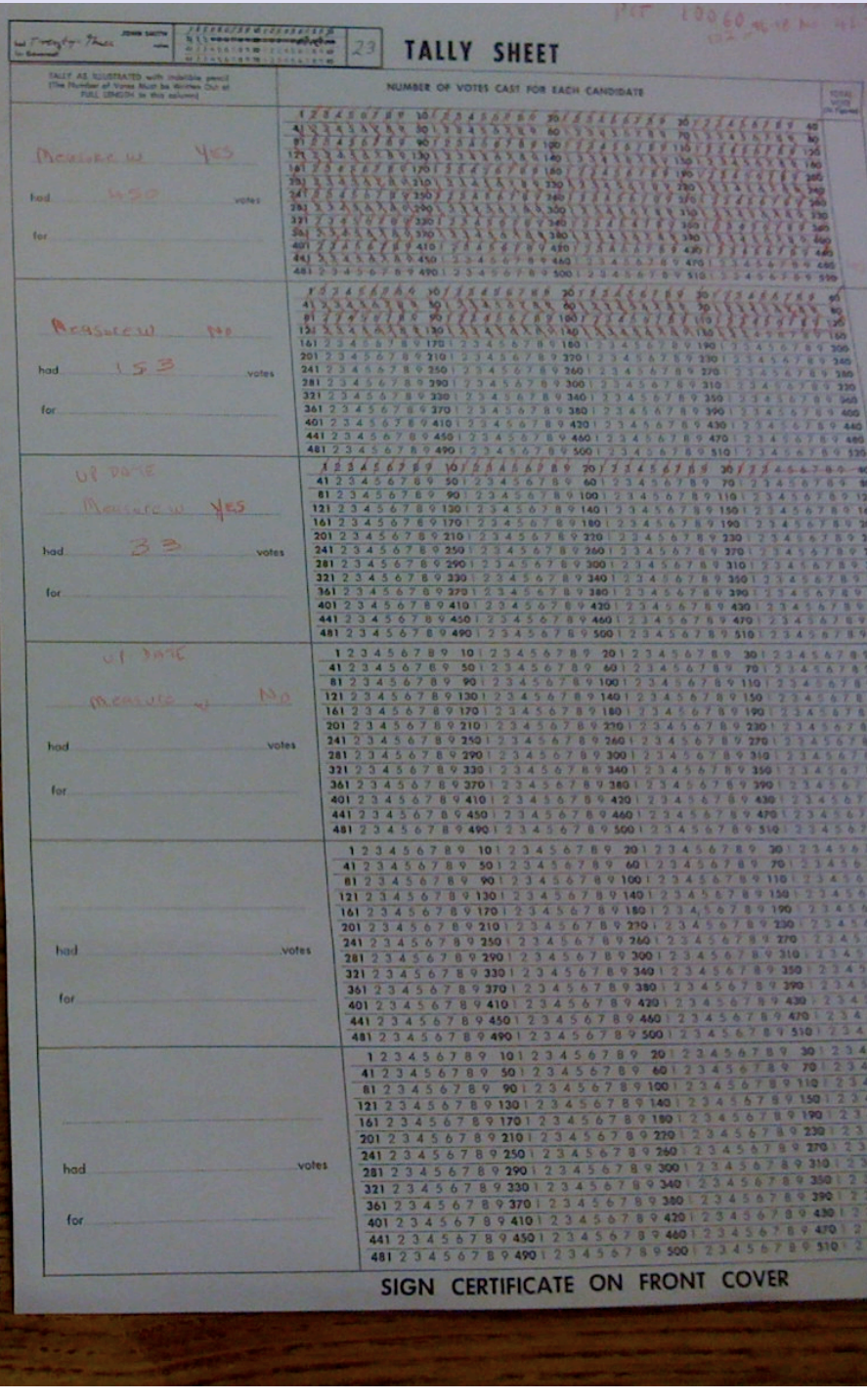
Cost per audited ballot ranged from nil to about $0.55.

2008 Yolo County, CA Measure W Audit




Voted Ballots
Hefty


TALLY SHEET
NUMBER OF VOTES CAST FOR EACH CANDIDATE
SIGN CERTIFICATE ON FRONT COVER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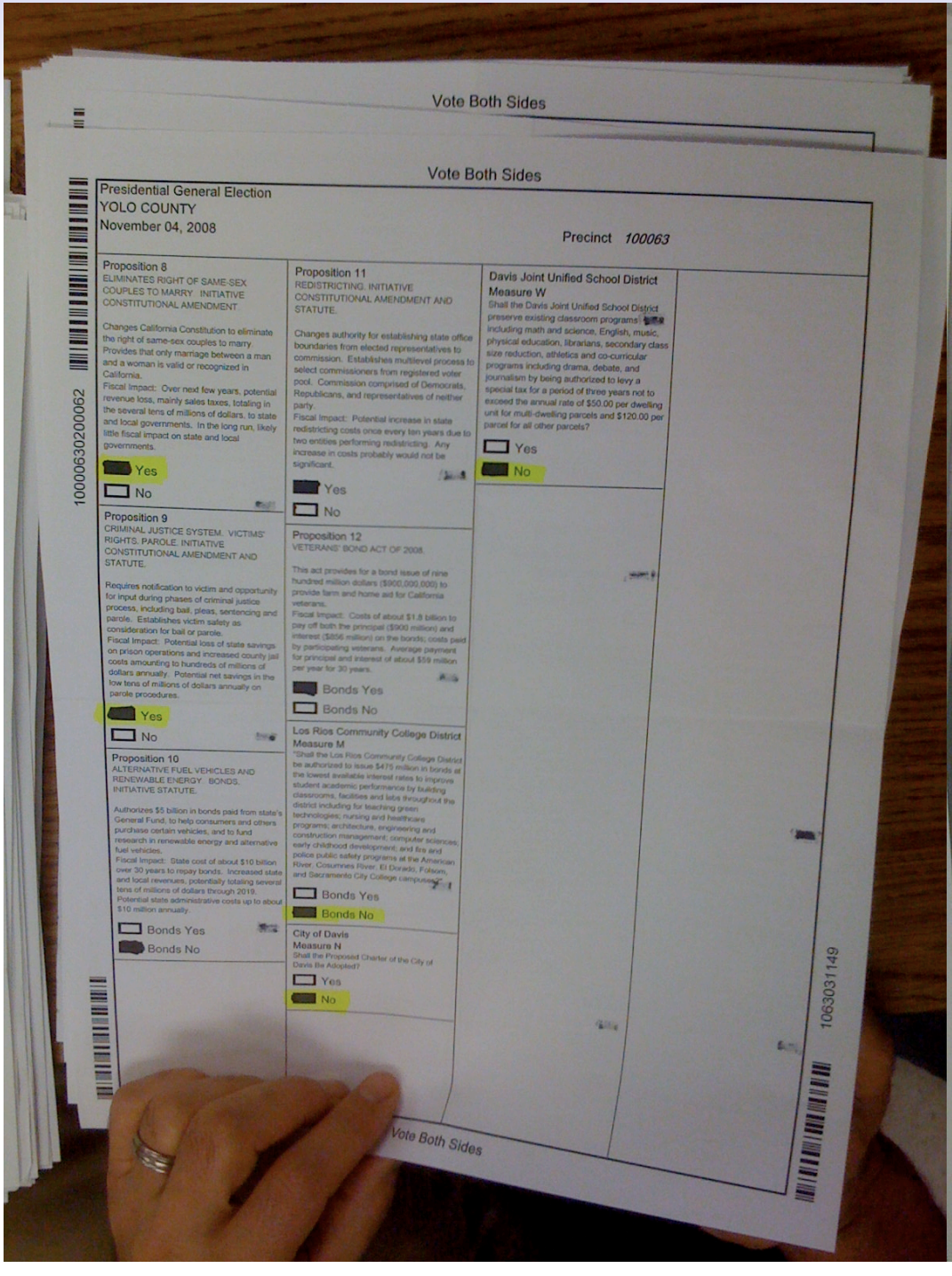
Vote Both Sides

Vote Both Sides

Presidential General Election
YOLO COUNTY
November 04, 2008

Precinct *100063*

Proposition 8
ELIMINATES RIGHT OF SAME-SEX COUPLES TO MARRY. INITIATIVE CONSTITUTIONAL AMENDMENT

Changes California Constitution to eliminate the right of same-sex couples to marry. Provides that only marriage between a man and a woman is valid or recognized in California.
Fiscal Impact: Over next few years, potential revenue loss, mainly sales taxes, totaling in the several tens of millions of dollars, to state and local governments. In the long run, likely little fiscal impact on state and local governments.

[x] Yes
[ ] No

Proposition 9
CRIMINAL JUSTICE SYSTEM. VICTIMS' RIGHTS. PAROLE. INITIATIVE CONSTITUTIONAL AMENDMENT AND STATUTE.

Requires notification to victim and opportunity for input during phases of criminal justice process, including bail, pleas, sentencing and parole. Establishes victim safety as consideration for bail or parole.
Fiscal Impact: Potential loss of state savings on prison operations and increased county jail costs amounting to hundreds of millions of dollars annually. Potential net savings in the low tens of millions of dollars annually on parole procedures.

[x] Yes
[ ] No

Proposition 10
ALTERNATIVE FUEL VEHICLES AND RENEWABLE ENERGY. BONDS. INITIATIVE STATUTE.

Authorizes $5 billion in bonds paid from state's General Fund, to help consumers and others purchase certain vehicles, and to fund research in renewable energy and alternative fuel vehicles.
Fiscal Impact: State cost of about $10 billion over 30 years to repay bonds. Increased state and local revenues, potentially totaling several tens of millions of dollars through 2019. Potential state administrative costs up to about $10 million annually.

[ ] Bonds Yes
[x] Bonds No

Proposition 11
REDISTRICTING. INITIATIVE CONSTITUTIONAL AMENDMENT AND STATUTE.

Changes authority for establishing state office boundaries from elected representatives to commission. Establishes multilevel process to select commissioners from registered voter pool. Commission comprised of Democrats, Republicans, and representatives of neither party.
Fiscal Impact: Potential increase in state redistricting costs once every ten years due to two entities performing redistricting. Any increase in costs probably would not be significant.

[x] Yes
[ ] No

Proposition 12
VETERANS' BOND ACT OF 2008.

This act provides for a bond issue of nine hundred million dollars ($900,000,000) to provide farm and home aid for California veterans.
Fiscal Impact: Costs of about $1.8 billion to pay off both the principal ($900 million) and interest ($856 million) on the bonds; costs paid by participating veterans. Average payment for principal and interest of about $59 million per year for 30 years.

[x] Bonds Yes
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Los Rios Community College District Measure M
"Shall the Los Rios Community College District be authorized to issue $475 million in bonds at the lowest available interest rates to improve student academic performance by building classrooms, facilities and labs throughout the district including for teaching green technologies; nursing and healthcare programs; architecture, engineering and construction management; computer sciences; early childhood development; and fire and police public safety programs at the American River, Cosumnes River, El Dorado, Folsom, and Sacramento City College campuses?"

[ ] Bonds Yes
[x] Bonds No

City of Davis Measure N
Shall the Proposed Charter of the City of Davis Be Adopted?

[ ] Yes
[x] No

Davis Joint Unified School District Measure W
Shall the Davis Joint Unified School District preserve existing classroom programs including math and science, English, music, physical education, librarians, secondary class size reduction, athletics and co-curricular programs including drama, debate, and journalism by being authorized to levy a special tax for a period of three years not to exceed the annual rate of $50.00 per dwelling unit for multi-dwelling parcels and $120.00 per parcel for all other parcels?

[ ] Yes
[x] No

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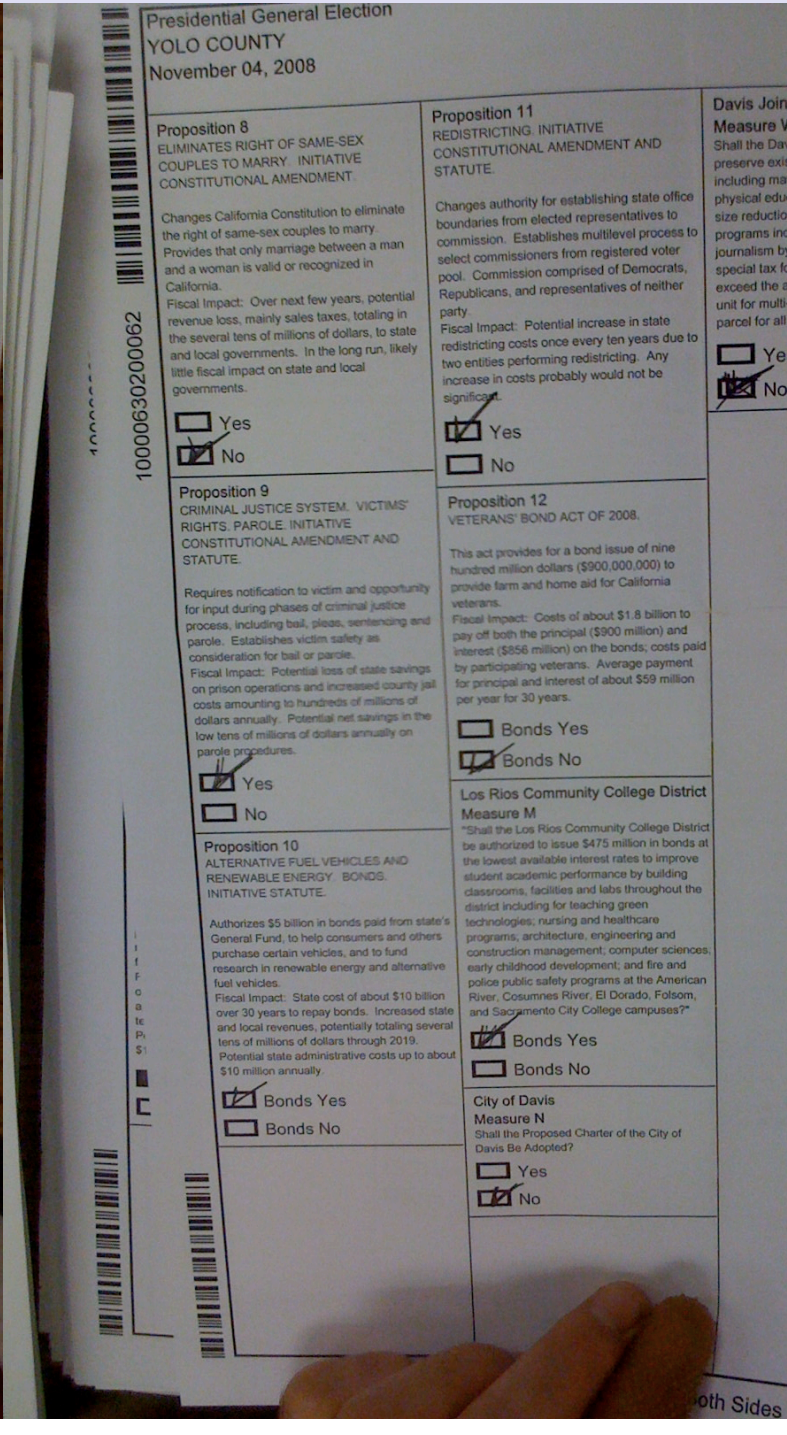
Vote Both Sides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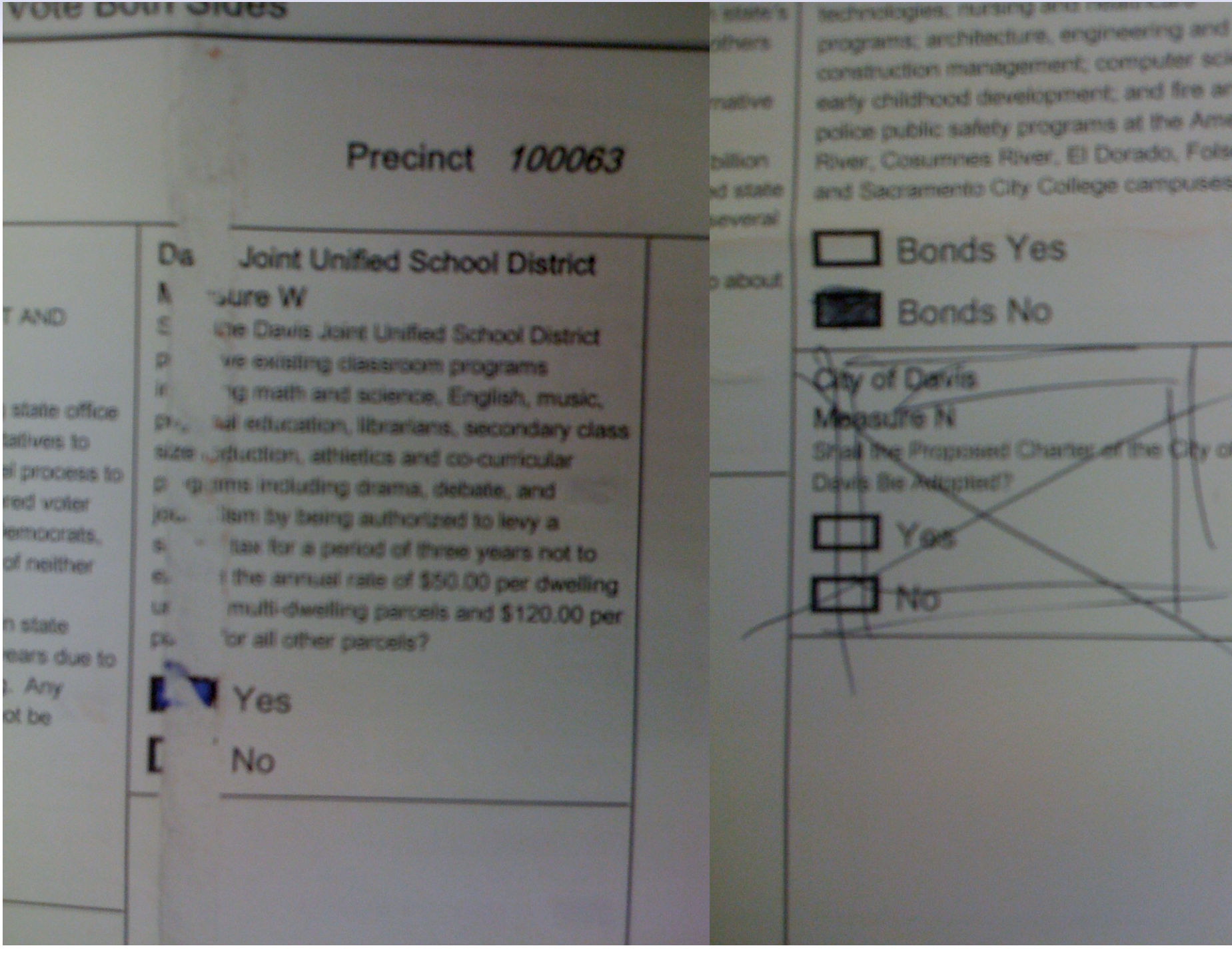
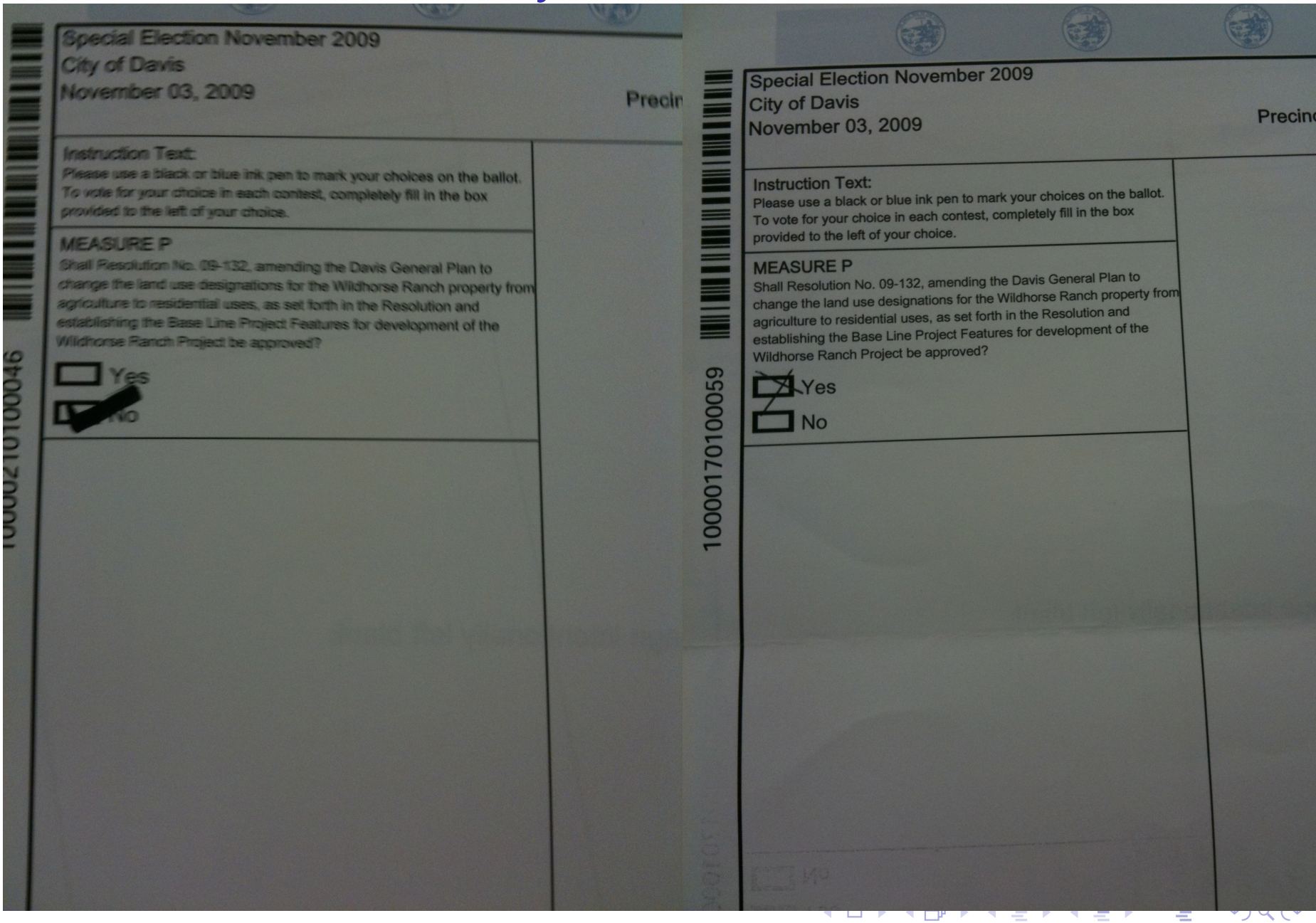
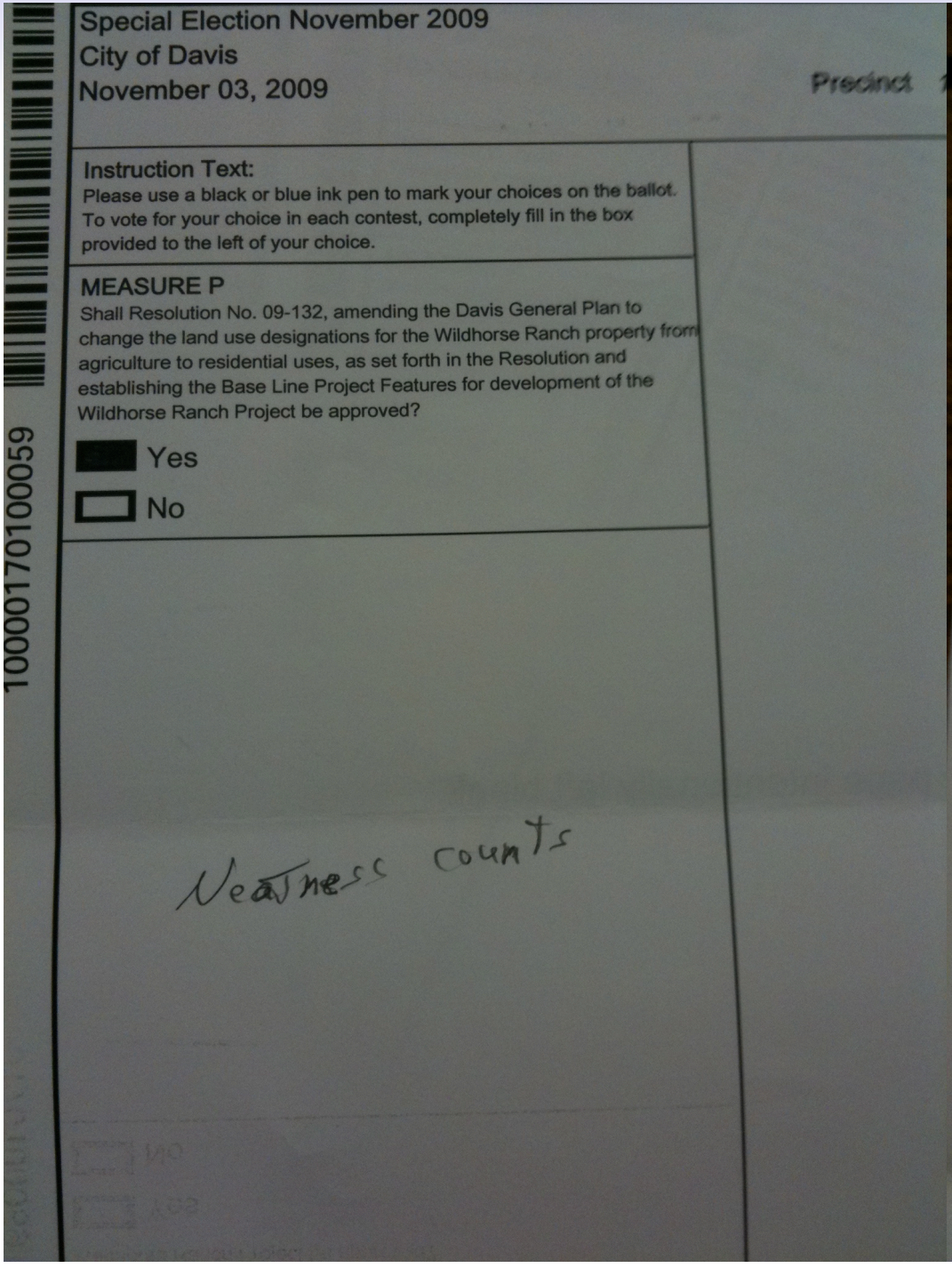

Vote Both Sides
Precinct 100063
Joint Unified School District
Yes
No
Bonds Yes
Bonds No
City of Davis
Yes
No

2009 Yolo County, CA Measure P Audit




Special Election November 2009
City of Davis
November 03, 2009

Precinct

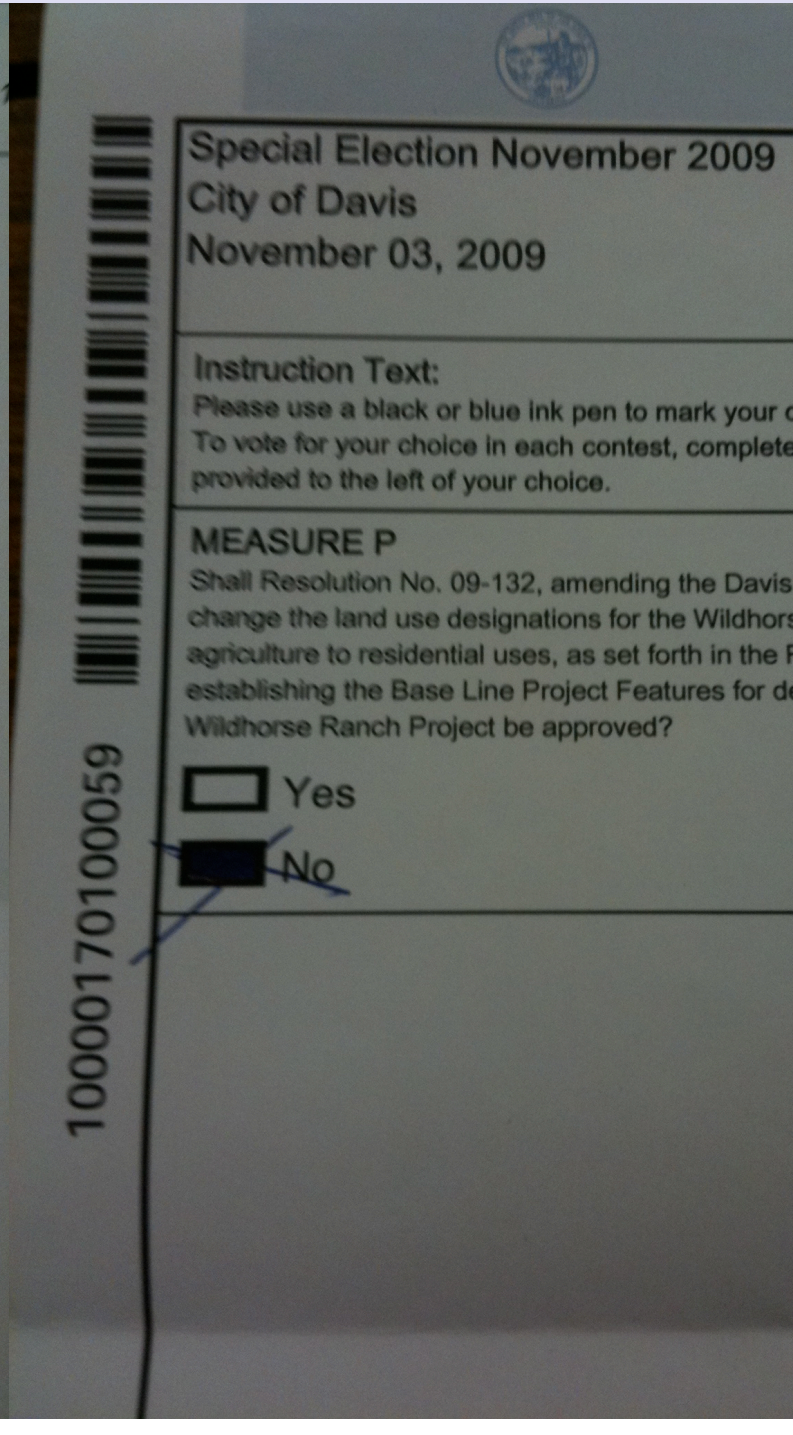
Instruction Text:
Please use a black or blue ink pen to mark your choices on the ballot. To vote for your choice in each contest, completely fill in the box provided to the left of your choice.

MEASURE P
Shall Resolution No. 09-132, amending the Davis General Plan to change the land use designations for the Wildhorse Ranch property from agriculture to residential uses, as set forth in the Resolution and establishing the Base Line Project Features for development of the Wildhorse Ranch Project be approved?

■ Yes
☐ No

Neatness counts

10000170100059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10000170100059

Yolo County Measure P, November 2009

Reg. voters	ballots	precincts	batches	yes	no
38,247	12,675	31	62	3,201	9,465

(VBM) and in-person (IP) ballots were tabulated separately (62 batches).

$U = 3.0235$.

For $\alpha = 10\%$, initial sample size 6 batches; gave 4 distinct batches, 1,437 ballots.

Ballot-level auditing would save *lots* of work

Can determine the initial sample size for a Kaplan-Markov ballot-level audit even though the cast vote records (CVRs) were not available.

For $\alpha = 10\%$ would need to look at CVRs for $n = 6$ ballots.

For $\alpha = 1\%$, $n = 12$ ballots.

C.f., 1,437 ballots for actual batch sizes.

Super-simple simultaneous audits

Goal

Truly simple audit rules that allow elections officials to confirm that the outcomes of most contests are right, with one (small) sample.

Risk-limiting: large chance of correcting any outcomes that are wrong—i.e., that disagree with the outcome full hand count of the audit trail would show. (Correct them by conducting a full hand count.)

Exploit statistical efficiency of *ballot-level auditing*, which compares CVR with human interpretation of individual ballots.

Spend some efficiency to buy logistic and computational simplicity.

Have to match CVRs to physical ballots. Requires new voting systems or *transitive auditing* using parallel systems (e.g., Clear Ballot Group, Humboldt ETP, TrueBallot) *a la* Calendrino et al. (2007)

Advantages of super-simple method

- Audit entire collection of contests with one simple random sample of ballots.
- Super simple: initial sample size is a constant—the sample size multiplier ρ—divided by the "diluted margin." ρ set once and for all: doesn't depend on any particulars of the contests, margins, etc.
- Audit expands if too many ballots with errors that overstate a margin by one vote, or any ballots that overstate a margin by two votes. Determining when to stop is simple.
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Ballot-level auditing

	contest				
	1	2	3	4	5
CVR	undervote	winner	loser	not on ballot	not on ballot
Hand	loser	loser	winner	not on ballot	not on ballot
overstatement	1	2	-2	0	0

Hypothetical CVR and hand interpretation of a ballot that contains three of five contests under audit. “Winner” and “loser” denote an apparent winner and an apparent loser, respectively. The maximum overstatement is two votes.

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CVR	undervote	winner	loser	winner	not on ballot
Hand	loser	overvote	loser	winner	not on ballot
overstatement	1	1	0	0	0

Hypothetical CVR and hand interpretation of a ballot that contains four of five contests under audit. “Winner” and “loser” denote an apparent winner and an apparent loser, respectively. In contest 3, the CVR and hand count found votes for one and the same apparent loser, and in contest 4, the CVR and hand count found votes for one and the same apparent winner. There are two overstatement errors, but the maximum overstatement is one vote.

New procedure requires picking risk and 2 parameters:

- simultaneous risk limit α. Might be set by legislation.
- error inflation factor $\gamma \geq 100\%$. Controls tradeoff between initial sample size and additional counting when the sample finds many overstatements. γ affects operating characteristics but not risk.
- error tolerance $\lambda < 100\%$. If rate of ballots in the sample with 1-vote maximum overstatements is no more than $\lambda\mu$ and there are no 2-vote overstatement, audit stops. λ affects operating characteristics but not risk.

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Super-simple simultaneous procedure

1. Pick risk limit $\alpha \in (0, 1)$, $\gamma \geq 100\%$, $\lambda < 100\%$
2. Calculate the sample-size multiplier ρ

$$\rho = \frac{-\log \alpha}{\frac{1}{2\gamma} + \lambda \log(1 - \frac{1}{2\gamma})}.$$

   For $\alpha = 10\%$, $\gamma = 110\%$ and $\lambda = 50\%$, $\rho = 15.2$. ρ doesn't depend on the audit data or particulars of the contests.
3. Calculate the diluted margin μ.
4. Audit simple random sample of at least $n = \lceil \rho/\mu \rceil$ ballots. If fewer than $n\lambda\mu$ of those have one-vote maximum overstatements and none has a two-vote overstatement, stop. Otherwise, Kaplan-Markov *P*-value determines when to stop.

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Examples

	$\lambda = 50\%$			$\lambda = 20\%$		
diluted	risk limit α			risk limit α		
margin μ	10%	5%	1%	10%	5%	1%
5%	305	396	609	139	180	277
2%	761	989	1521	346	450	691
1%	1521	1978	3041	691	899	1382
0.5%	3041	3956	6081	1382	1798	2764
multiplier ρ	15.20	19.78	30.40	6.91	8.99	13.82

Initial sample sizes *n* and multipliers ρ for $\gamma = 110\%$. Column 1: diluted margin of victory μ. Columns 2–4: *n* for various risk limits if the audit is to stop when the percentage of ballots in the sample that overstate a margin by 1 vote is not more than 50% of the diluted margin. Columns 5–7: *n* for various simultaneous risk limits if audit is to stop when the percentage of ballots in the sample that overstate a margin by 1 vote is not more than 20% of the diluted margin. Last row: sample sizes *n* are equal to these "multipliers" divided by diluted margin μ.

Conclusions

Very simple formula for initial sample for risk-limiting audit. Allows audit to stop if, in the initial sample, rate of 1-vote maximum overstatements is at most a pre-specified fraction of the margin and there are no 2-vote overstatements.

Method requires choosing 2 numbers that affect operating characteristics but not risk.

Simple but somewhat inefficient: More ballots have to be counted by hand than if sharper bounds were used, but those methods require far more complex math.

Ballot-level audits are so efficient that total cost still low.

Secret sauce

To implement ballot-level audits on a wide scale may require changes to vote tabulation systems: have to associate individual cast vote records (CVRs) with individual physical ballots.

Auditing using an unofficial vote tabulation system that does produce CVRs—such as those of Clear Ballot Group, the Humboldt Transparency Project, or TrueBallot—and confirming transitively that the apparent outcome is correct, might be the best interim option. (See Calendrino et al. 2007)

What do we need for efficient audits?

Laws that allow/require risk-limiting audits, but mostly . . .

Data plumbing:

Structured, small batch data export from VTSs.

A way to associate individual CVRs with physical ballots.

Reducing counting effort is mostly about reducing batch sizes.

New York's Recent Reforms

Moved to precinct-count optically scanned paper ballots.

Introduced audit laws, starting with 3% of machines (scanners).

Irreconcilable differences between hand count and machine count can lead to counting more ballots by hand: 5%, 12%, or all.

NY SD 7

- Balance of power in NY Senate: Either 31 seats for each party, or 32 for Republicans.
- Reported margin of 451 votes (0.5%) for Republican candidate Martins.
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- 7 of 249 of machines audited at random (3%).
- 3 of 7 (i.e., 43%) showed errors. Net error favored the apparent winner.
- Republicans: the errors were “reconciled”: Machines are fine. Democrats disagree.
- Judge Warshawsky: “In my opinion, reconcilable would be ‘Is there a clear reason why the deficiency occurred?’ ”
- Is “the machine was mis-programmed” a clear reason?
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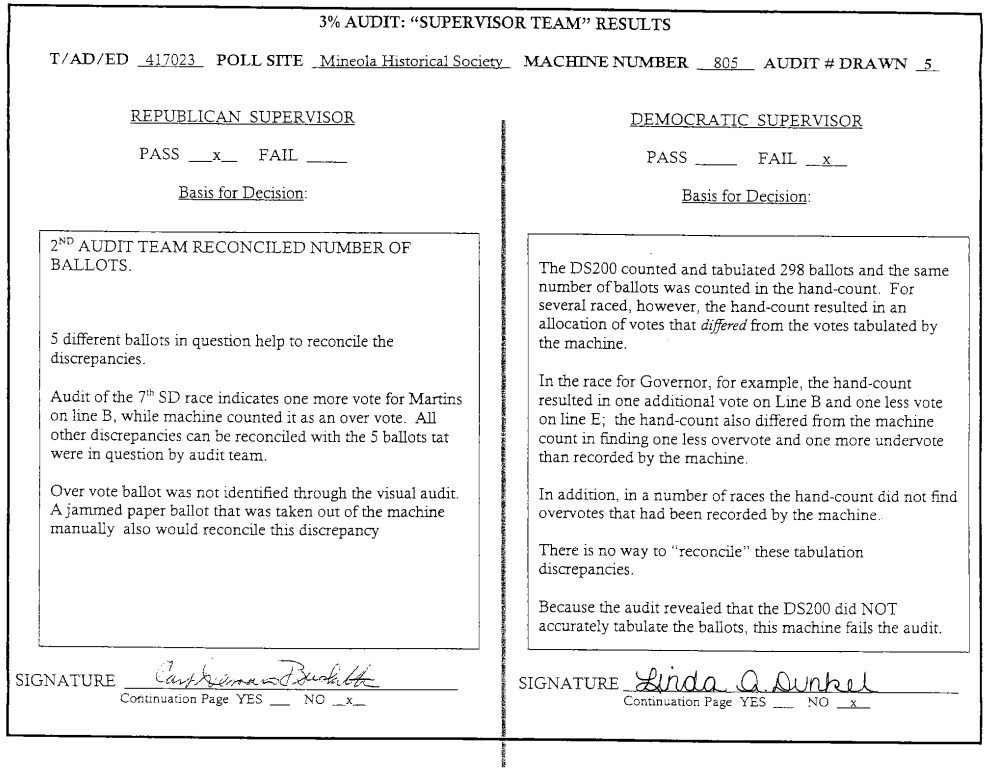
- "Clear reason" is irrelevant for whether the apparent outcome of the contest is correct. Size of the difference matters.

Example NY SD7 Audit report



3% AUDIT: "SUPERVISOR TEAM" RESULTS

T/AD/ED 417023 POLL SITE Mineola Historical Society MACHINE NUMBER 805 AUDIT # DRAWN 5

REPUBLICAN SUPERVISOR

PASS __x__ FAIL ____

Basis for Decision:

2ND AUDIT TEAM RECONCILED NUMBER OF BALLOTS.

5 different ballots in question help to reconcile the discrepancies.

Audit of the 7th SD race indicates one more vote for Martins on line B, while machine counted it as an over vote. All other discrepancies can be reconciled with the 5 ballots tat were in question by audit team.

Over vote ballot was not identified through the visual audit. A jammed paper ballot that was taken out of the machine manually also would reconcile this discrepancy

SIGNATURE [signature]

Continuation Page YES ___ NO _x_

DEMOCRATIC SUPERVISOR

PASS ____ FAIL __x__

Basis for Decision:

The DS200 counted and tabulated 298 ballots and the same number of ballots was counted in the hand-count. For several raced, however, the hand-count resulted in an allocation of votes that *differed* from the votes tabulated by the machine.

In the race for Governor, for example, the hand-count resulted in one additional vote on Line B and one less vote on line E; the hand-count also differed from the machine count in finding one less overvote and one more undervote than recorded by the machine.

In addition, in a number of races the hand-count did not find overvotes that had been recorded by the machine.

There is no way to "reconcile" these tabulation discrepancies.

Because the audit revealed that the DS200 did NOT accurately tabulate the ballots, this machine fails the audit.

SIGNATURE Linda A. Dunkel

Continuation Page YES ___ NO _x_

HO 13-24 machine 952:

It was evident that the discrepancy between the audit (manual count) of the votes reflected on the ballots in the ballot box, and the votes reflected on the scanner result tape, are attributable to the two additional ballots found in the ballot box. The disparity between the manual count, and the scanner result tape, are precisely equal to the votes reflected when counting all off [sic] the ballots in the ballot box – including the two additional ballots. This is not "scanner error," but is instead attributable to any one of a number of alternative possibilities . . . The presence of more ballots in the ballot box does not demonstrate that the scanner has "failed," merely that the machine operated as it was designed to do – but with the result that some number (in this case, two) ballots were not scanned. . . . PASS

More

2-4059 Machine 104

1 additional ballot found and explains all discrepancies. . . . PASS

H18-12 Machine 259

One additional ballot was counted by the scanner than was found in the bin . . . PASS

Proffered testimony

The audit results not surprising even if a full hand count would show Mr. Johnson to be the winner.

Substantial possibility that the machine with the largest error was not one of the machines that was audited. 97% chance that auditing 7 of 249 machines won't check the machine with the largest error.

Average of less than two errors per machine could account for the apparent margin of about 450 votes.

Average of one error per 200 ballots could account for the apparent margin.

Proffer, contd.

Not a surprising level of error in precinct-count optically scanned ballots. Consistent with the errors the audit did find, within the statistical variability expected from “the luck of the draw.”

Large potential for error: the 242 unaudited machines could hold enough error to account for the apparent margin 186 times over. Sixty-six of the 242 unaudited machines could *individually* hold enough error to account for the apparent margin.

Proffer, contd.

Substantial chance that a 3% or 8% audit would find little or no error even if Sen. Johnson is the true winner.

If 30 of the 249 machines have errors of 15 votes or more—enough to account for the apparent margin—chance the 3% audit would have found any of those machines is under 60%.

If 20 of the 249 machines have errors of 23 votes or more–enough to account for the apparent margin–chance the 3% audit would have found any of those machines is under 45%.

If 20 of the 242 unaudited machines have errors of 23 votes or more (enough to account for the apparent margin) and an additional 5% of the machines are audited, chance the additional audit would find any of those 20 is under 69%.

Proffered testimony, contd.

Margin is so small compared to the possible errors that very large percentage of machines must be audited to give strong evidence that Mr. Martins is indeed the winner.

3% is not sufficient.

8% is not sufficient.

To have 90% statistical confidence that Mr. Martins won requires auditing a minimum of 90% of the machines selected randomly: an additional 218 machines.

This is true if the audit finds that those 218 machines have counted perfectly. If the audit of those 218 machines found many errors, still more machines would have to be audited.