Statistics 240 Lecture Notes

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1 Assignment 1

1. Identify the question and source of data for your term project.

2. Let A, B and C be sets. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

3. Let A and B be sets. Show that $A - B = \emptyset$ implies $A \subset B$.

4. Show that for any sets A, B, C, D, $(A \bigotimes B) \cap (C \bigotimes D) = (A \cap C) \bigotimes (B \cap D)$.

5. Show that for any function f with domain $\mathcal{X}$, if $A, B \subset \mathcal{X}$, then $f(A \cap B) = fA \cap fB$, and that $f(A \cup B) = fA \cup fB$.

6. Let f be a function with co-domain $\mathcal{Y}$, and $A, B \subset \mathcal{Y}$. Does $f^{-1}(A \cap B) = f^{-1}A \cap f^{-1}B$? Does $f^{-1}(A \cup B) = f^{-1}A \cup f^{-1}B$?

7. Let f have domain $\mathcal{X}$ and co-domain $\mathcal{Y}$, and suppose that $A \subset \mathcal{X}$ and $B \subset \mathcal{Y}$. Does $f^{-1}(f(A)) = A$? Does $f(f^{-1}B) = B$?

8. Let $\mathcal{G}$ be a group with identity e. Show that $ae = (a^{-1})^{-1} = a$. (That is, show that e is not only the identity from the left, it is also the identity from the right, and that if $a^{-1}a = e$, then $aa^{-1} = e$.)

9. Let $a, b, c, d \in F$, where F is a field. Show that if $b, d \neq 0$, then $a/b + c/d = (ad + bc)/bd$.

10. Show that $A = \{0, 1, 2, \cdots, p-1\}$ with p prime is a field, if addition and multiplication are defined modulo p. What breaks down if p is not prime? For $p = 7$, show that the multiplicative inverse of 2 is 4.