

# 2008 Risk-Limiting Audits in California

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**Abstract:** Post-election audits—comparisons of reported totals with hand counts of a trustworthy audit trail in a random sample of batches—can limit the risk of certifying an incorrect electoral outcome. The risk is the maximum chance the audit stops short of a full hand count when a full hand count would show that the outcome is wrong.

Risk-limiting audits can be built as sequential tests: Data are collected. If they give strong evidence that the outcome is right, the audit stops. Otherwise, more data are collected. Eventually, the audit stops or there has been a full hand count. Limiting the risk requires recognizing that each stage of the audit is a new opportunity to err.

There have been four risk-limiting audits, all in California in 2008: Marin County (a small measure in February requiring a supermajority and a county-wide measure in November), Santa Cruz County (County Supervisor, District 1, November), and Yolo County (bond measure). Several sampling techniques were tested. The audits ensured at least a 75% chance of a full hand count if that would change the outcome.

Many lessons were learned. Clear, precise and timely communication between the auditors and the elections officials is key. The biggest barrier is the inability of current election management systems (vote tabulation systems) to export data in a useful, machine-readable format. Insisting that vendors provide better data export should be a high priority. Adopting standard terminology and data formats would be extremely helpful.

## Machine (Voting System) Counting

- Want to count votes by machine: saves time and money (or so we are told).
- Machine counts are subject to various kinds of error. (So are hand counts, but they're the gold standard. Progress on accuracy, too.)
- Counting errors  $\Rightarrow$  risk that machines name the wrong winner.

## Risk-Limiting Audits

If the outcome is wrong, there's a at least a [pre-specified] chance of a full manual count

no matter what caused the outcome to be wrong, even if an evil adversary built the hardware and wrote the software.

The *risk* is the chance that there won't be a full hand count when a full hand count would show that the apparent outcome is wrong.

Essential that voters create complete, durable, accurate audit trail.

Essential that voting systems enable auditors to access reported results (total ballots, counts for each candidate, registered voters) in auditable batches.

Essential to select batches at random, *after* the results are posted. (Can supplement with “targeted” samples.)

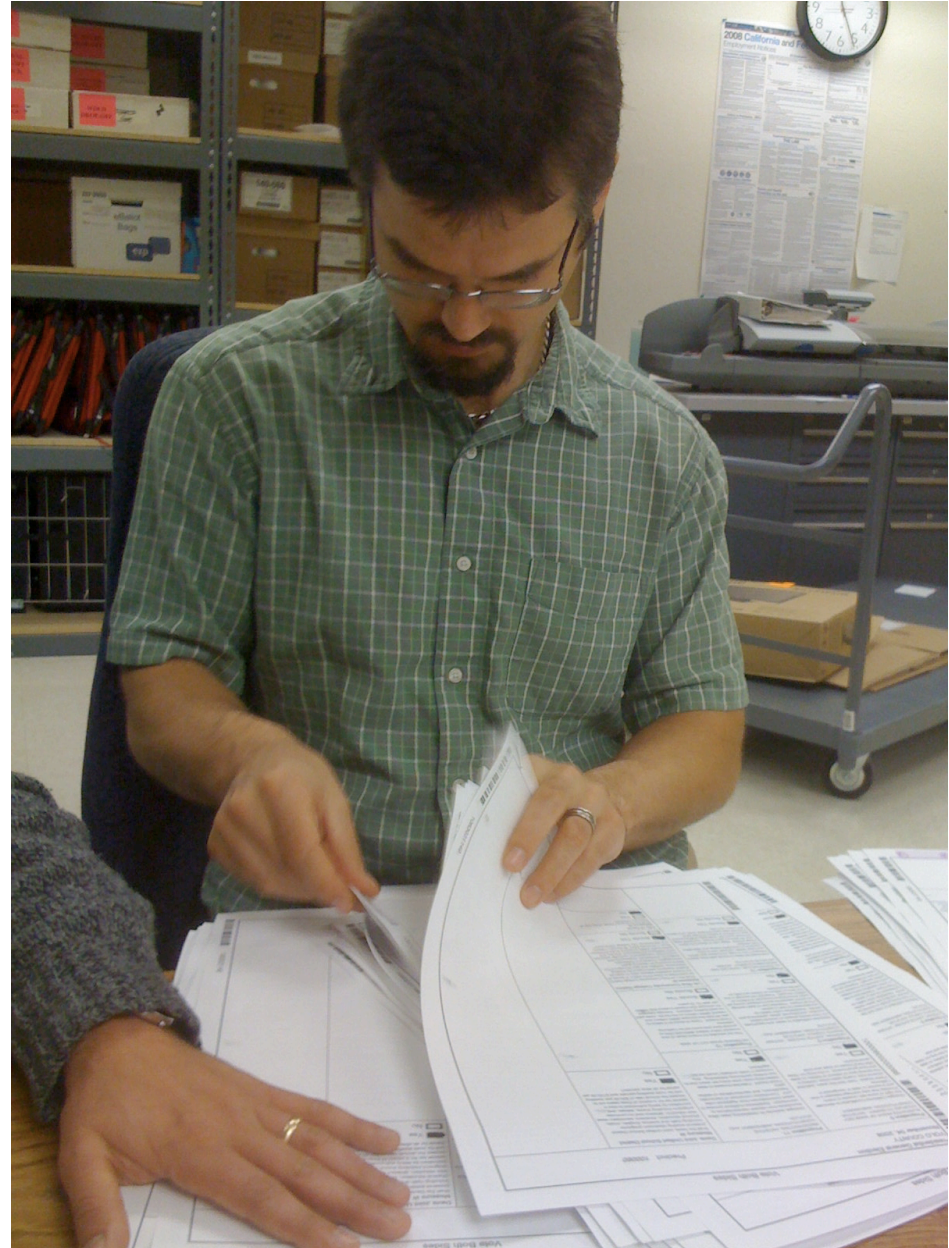
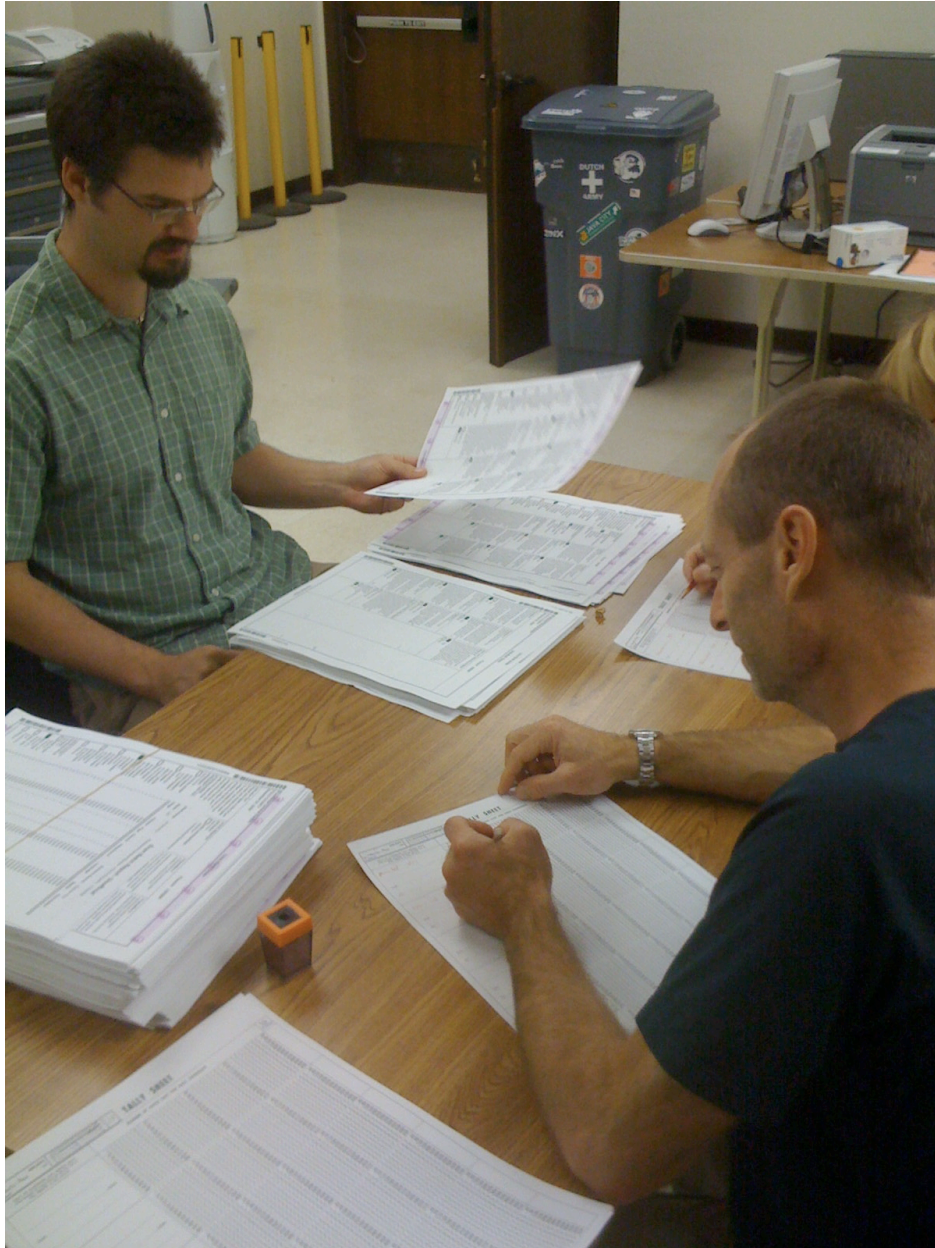
Need a plan for dealing with discrepancies, possibly leading to full count. “Explanation” is not enough.

Current audit laws do not limit risk.

Process audits vs. materiality audits.

# 2008 Yolo County, CA Measure W Audit











23 TALLY SHEET

TALLY AS ILLUSTRATED with indelible pencil (The Number of Votes must be Written Out in FULL LENGTH in this column)

NUMBER OF VOTES CAST FOR EACH CANDIDATE

TOTAL (Sum of Figures)

had **450** votes

for **450**

had **153** votes

for **153**

had **33** votes

for **33**

had **32** votes

for **32**

had **483** votes

for **483**

had **153** votes

for **153**

SIGN CERTIFICATE ON FRONT COVER

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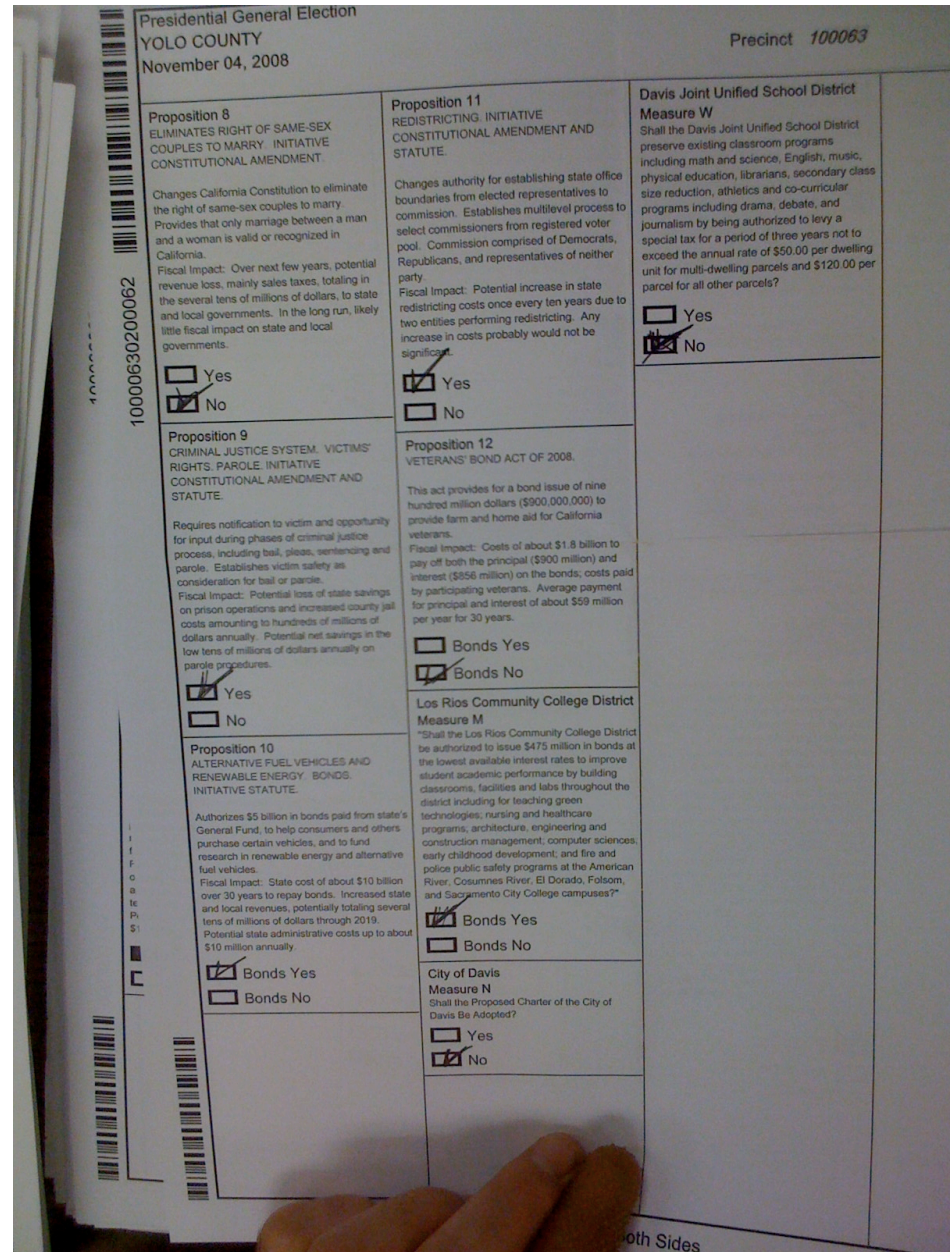
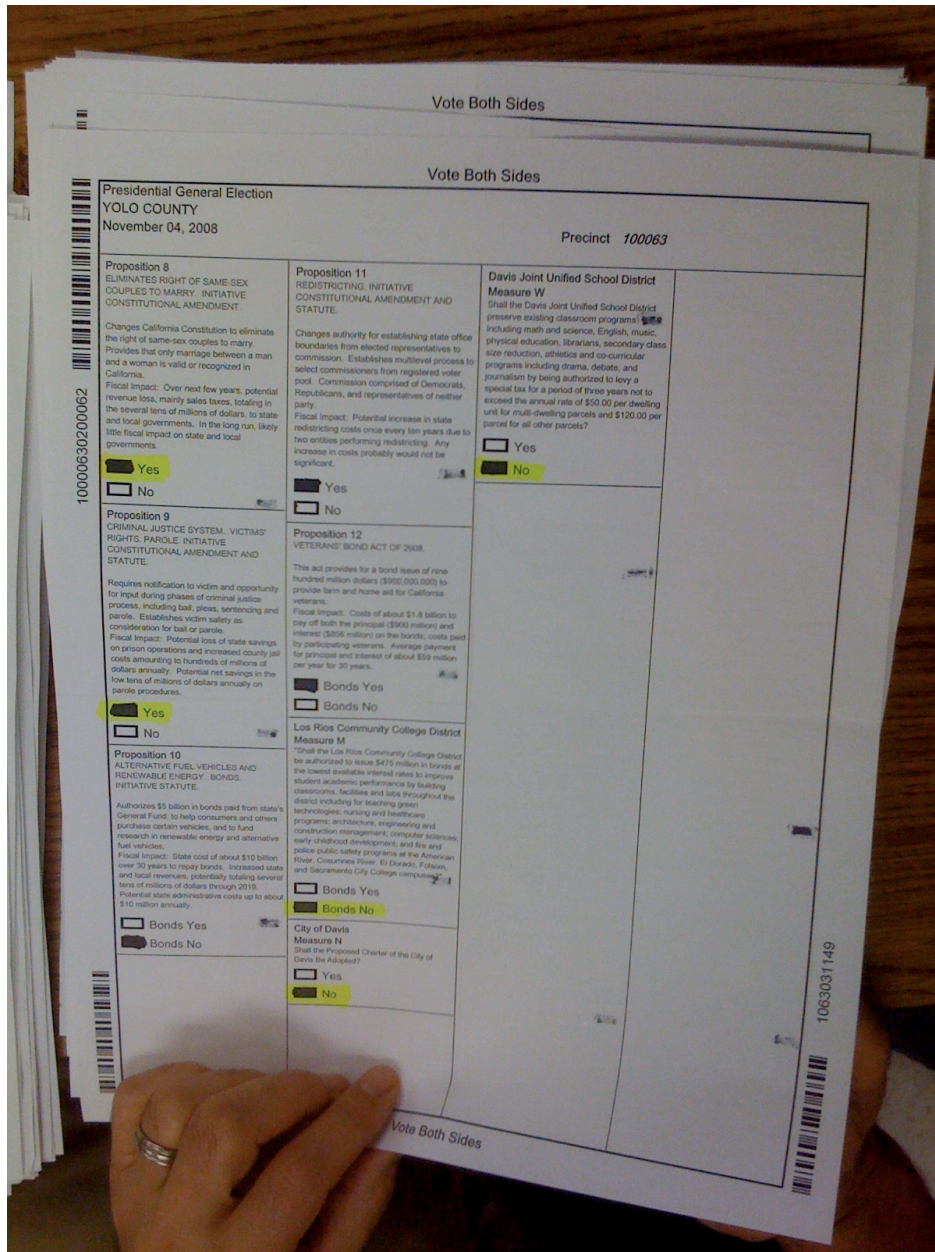
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Presidential General Election  
YOLO COUNTY  
November 04, 2008

Precinct 100063

**Proposition 8**  
ELIMINATES RIGHT OF SAME-SEX  
COUPLES TO MARRY. INITIATIVE  
CONSTITUTIONAL AMENDMENT

Changes California Constitution to eliminate the right of same-sex couples to marry. Provides that only marriage between a man and a woman is valid or recognized in California.  
Fiscal Impact: Over next few years, potential revenue loss, mainly sales taxes, totaling in the several tens of millions of dollars, to state and local governments. In the long run, likely little fiscal impact on state and local governments.

Yes  
 No

**Proposition 9**  
CRIMINAL JUSTICE SYSTEM VICTIMS' RIGHTS PAROLE INITIATIVE CONSTITUTIONAL AMENDMENT AND STATUTE

Requires notification to victim and opportunity for input during phases of criminal justice process, including bail, plea, sentencing and parole. Establishes victim safety as consideration for bail or parole.  
Fiscal Impact: Potential loss of state savings on prison operations and increased county jail costs amounting to hundreds of millions of dollars annually. Potential net savings in the low tens of millions of dollars annually on parole procedures.

Yes  
 No

**Proposition 10**  
ALTERNATIVE FUEL VEHICLES AND RENEWABLE ENERGY BONDS INITIATIVE STATUTE

Authorizes \$5 billion in bonds paid from state's General Fund, to help consumers and others purchase certain vehicles, and to fund research in renewable energy and alternative fuel vehicles.  
Fiscal Impact: State cost of about \$10 billion over 30 years to repay bonds. Increased state and local revenues, potentially totaling several tens of millions of dollars through 2019. Potential state administrative costs up to about \$10 million annually.

Bonds Yes  
 Bonds No

**Proposition 11**  
REDISTRICTING INITIATIVE CONSTITUTIONAL AMENDMENT AND STATUTE

Changes authority for establishing state office boundaries from elected representatives to commission. Establishes multilevel process to select commissioners from registered voter pool. Commission comprised of Democrats, Republicans, and representatives of neither party.  
Fiscal Impact: Potential increase in state redistricting costs once every ten years due to two entities performing redistricting. Any increase in costs probably would not be significant.

Yes  
 No

**Proposition 12**  
VETERANS' BOND ACT OF 2008

This act provides for a bond issue of nine hundred million dollars (\$900,000,000) to provide farm and home aid for California veterans.  
Fiscal Impact: Costs of about \$1.8 billion to pay off both the principal (\$900 million) and interest (\$900 million) on the bonds, costs paid by participating veterans. Average payment for principal and interest of about \$50 million per year for 30 years.

Bonds Yes  
 Bonds No

**Los Rios Community College District Measure M**

Shall the Los Rios Community College District be authorized to issue \$475 million in bonds at the lowest available interest rates to improve student academic performance by building classrooms, facilities and labs throughout the district including for teaching green technologies, nursing and healthcare programs, architecture, engineering and construction management, computer sciences, early childhood development, and fire and police public safety programs at the American River, Cosumnes River, El Dorado, Folsom, and Sacramento City College campuses?

Bonds Yes  
 Bonds No

**City of Davis Measure N**

Shall the Proposed Charter of the City of Davis Be Adopted?

Yes  
 No

**Davis Joint Unified School District Measure W**

Shall the Davis Joint Unified School District preserve existing classroom programs including math and science, English, music, physical education, librarians, secondary class size reduction, athletics and co-curricular programs including drama, debate, and journalism by being authorized to levy a special tax for a period of three years not to exceed the annual rate of \$50.00 per dwelling unit for multi-dwelling parcels and \$120.00 per parcel for all other parcels?

Yes  
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Yes  
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Precinct 100063

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**Da** Joint Unified School District  
**M** Measure W  
**E** The Davis Joint Unified School District  
**p** improve existing classroom programs  
**i** in math and science, English, music,  
**p** physical education, librarians, secondary class  
**s** size reduction, athletics and co-curricular  
**p** programs including drama, debate, and  
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## Selecting batches at random

Software pseudo-random number generators:  
not transparent, hackable.

One ticket per precinct:  
hard to verify; hard to mix (Vietnam draft).

2008 Oregon law uses one ticket per precinct; allows  
selections before publishing election results.

10-sided dice (Marin County) [Roll 1] [Roll 2]

Ping-pong balls (Alameda County) [Static] [Tumbling]

Alameda has 1204 precincts. Pick 1s digit, 10s, 100s.  
If result is between 205 and 999, stop.  
Else, remove 2–9 & pick 1000s digit.

Unintended consequences?



## Logistic tradeoff

Errors and time for dice rolling, look-up tables (especially for PPS sampling).

Hybrid selection

November 2008 Marin & Santa Cruz audits)

Roll 10-sided dice to get a 6-digit seed.

Use “good” open source PRNG to generate a sequence of numbers from the seed in a reproducible way.

(Used the Mersenne Twister implemented in R)

## General principles

Margin small  $\Rightarrow$  less error required to change outcome.

Sample too small  $\Rightarrow$  sample can be likely to find few or no errors, even if outcome is wrong.

Sample big (compared with margin & error bounds)  $\Rightarrow$  likely to see big discrepancies in the sample if machines named wrong winner.

No look, no see: absence of evidence is not evidence of absence.

## Rigorous statistical audit

If, for every way the outcome could be wrong, it would the audit is very likely to have found more error than it did find, stop.

Otherwise, keep counting.

If the audit stops without a full hand count, either the correct winner was named, or something very unlikely happened.

## Complete procedure says:

- how many batches to audit initially
- given the discrepancies in the audit sample, whether to stop or to expand the audit
- eventually declares “stop” or “full hand count.”
- guaranteed minimum chance of full hand count if outcome is wrong, e.g., 90%

Only one basic approach so far does that.



## Basic Idea

1. Pick the min chance  $1 - \alpha$  of full manual count when result is wrong (the risk is  $\alpha$ ). Pick the max # of stages  $S$ . Allocate risk to stages:  $\alpha_1, \dots, \alpha_S$ .
2. Define batches and strata. Choose sampling scheme. Define “error.” Set  $s = 1$ .
3. Draw sample and audit.
4. If, on the assumption that the outcome is wrong, the chance of seeing “so little” error is less than  $\alpha_s$ , stop. Otherwise, if  $s = S$ , count everything; but if  $s < S$ , increment  $s$  and go back to step 3.

## 5 February 2008 Marin County Measure A

**First election ever audited to attain target level of confidence in the result.**

Audited to attain 75% confidence that a full manual recount would find the same outcome.

Required 2/3 majority to pass. Margin 298 votes.

3 strata: in-precinct, VBM, provisionals

Confirmed outcome at no more than 25% risk (quite conservative)

## Marin Measure A audit timeline

Milestone	Date
Election day	5 February
Polling place results available	7 February
Random selection of polling place precincts	14 February
VBM results available	20 February
Random selection of VBM precincts	20 February
Hand tally complete	20 February
Provisional ballot results available	29 February
Computations complete	3 March

### Costs:

\$1,501, including salaries and benefits for 4 people tallying the count, a supervisor, support staff to print reports, resolve discrepancies, transport ballots and locate and retrieve VBM ballots from the batches in which they were counted.

\$0.35 per ballot audited.  $1\frac{3}{4}$  days.

## Marin Measure B and Santa Cruz Supervisor District 1, November 2008

Used PPS sampling and trinomial bound.

County	Ballots	Winner	Loser	Prcts	Batches	Batches Audited	Ballots Audited	% Ballots Audited
Marin	121,295	51%	35%	189	544	14	3,347	3%
SC	26,655	45%	37%	76	152	16	7,105	27%

Marin: no discrepancies.

Santa Cruz: “taints” 0.036, 0.007, -0.002, -0.003, -0.005, -0.007, -0.012; twelve were 0.

Miscommunication about provisional ballots in Santa Cruz; treated as error.



## Yolo County Measure W, November 2008

Davis school bond. Required simple majority. Used SRS.

batches	yes	no	undervote	overvote	margin
114	25,297	8,118	3,001	2	17,179

Stop if no batch has more than 14 overstatements.

Assumed “small” batches were entirely in error; sampled from remaining 95.

Counted about 2,500 ballots by hand on 17 November 2008.  
1 extra “yes” and 1 extra “no.”

Logistical issues: stratification, etc.

Samples for different counties drawn independently: stratified.

VBM, absentee & provisional ballots not counted right away.

Makes sense to start with a uniform sampling rate, then escalate as necessary.

## Recap

- Auditing laws that address the problem fall short
- Good audits can limit the risk of certifying a wrong outcome
- There are practical ways to conduct risk-limiting audits
- Data plumbing is crucial! First step for any jurisdiction.

## References

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Verified Voting <http://verifiedvoting.org>

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More voting-related links: <http://statistics.berkeley.edu/~stark/Vote/index.htm>