


## A34 Answers to Selected Problems

3. Multinomial,  $n = 10$ ,  $p_1 = p_2 = p_3 = 1/3$
7.  $f_{XY}(x, y) = \alpha\beta \exp[-\alpha x - \beta y]$ ;  $f_X(x) = \alpha \exp[-\alpha x]$ ,  $f_Y(y) = \beta \exp[-\beta y]$
9. a.  $f_X(x) = 3(1 - x^2)/4$ ,  $-1 \leq x \leq 1$ ,  $f_Y(y) = 3\sqrt{1 - y^2}/2$ ,  $0 \leq y \leq 1$   
 b.  $f_{X|Y}(x|y) = 1/(2\sqrt{1 - y^2})$ ,  $f_{Y|X}(y|x) = 1/(1 - x^2)$
11. ~~1/9~~ 
13.  $p(0) = 1/2$ ,  $p(1) = p(2) = 1/4$
15. a.  $c = 3/2\pi$                       c.  $\frac{2\sqrt{2} - 1}{2\sqrt{2}}$
- d.  $f_Y(y) = \frac{3}{2}(1 - y^2)$ ,  $-1 \leq y \leq 1$   
 $f_X(x) = \frac{3}{2}(1 - x^2)$ ,  $-1 \leq x \leq 1$   
 $X$  and  $Y$  are not independent.
- e.  $f_{Y|X}(y|x) = \frac{\sqrt{1 - x^2 - y^2}}{\pi(1 - x^2)}$   
 $f_{X|Y}(x|y) = \frac{\sqrt{1 - x^2 - y^2}}{\pi(1 - y^2)}$
17. b.  $f_X(x) = 1 - |x|$ ,  $-1 \leq x \leq 1$ ;  $f_Y(y) = 1 - |y|$ ,  $-1 \leq y \leq 1$   
 c.  $f_{X|Y}(x|y) = 1/(2 - 2|y|)$ ,  $1 - |y| \leq x \leq 1 + |y|$   
 $f_{Y|X}(y|x) = 1/(2 - 2|x|)$ ,  $1 - |x| \leq y \leq 1 + |x|$
19. a.  $\beta/(\alpha + \beta)$                       b.  $\beta/(2\alpha + \beta)$
23. Binomial ( $m, pr$ )
29.  $h(x, y) = \lambda\mu e^{-\lambda x} e^{-\mu y} [1 + \alpha(1 - 2e^{-\lambda x})(1 - 2e^{-\mu y})]$
33. a.  $f_{\Theta|N}(\theta|n) = n(n + 1)\theta(1 - \theta)^{n-1}$
43.  $f_S(s) = s$  for  $0 \leq s \leq 1$  and  $= 2 - s$  for  $1 \leq s \leq 2$
49.  $\lambda e^{-\lambda S/2} - \lambda e^{-\lambda S}$                       53.  $5/9$
55.  $f_{XY}(x, y) = (x^2 + y^2)^{-1/2}$ ,  $x^2 + y^2 \leq 1$
57.  $x_1 = y_1$ ;  $x_2 = -y_1 + y_2$
61.  $f_{UV}(u, v) = \frac{1}{bd} f_{XY}\left(\frac{u - a}{b}, \frac{v - c}{d}\right)$
63. a.  $f_{UV}(u, v) = \frac{1}{2} f_{XY}\left(\frac{u + v}{2}, \frac{u - v}{2}\right)$  where  $U = X + Y$ ,  $V = X - Y$   
 b.  $f_{UV}(u, v) = \frac{1}{2|v|} f_{XY}((uv)^{1/2}, (u/v)^{1/2})$  where  $U = XY$ ,  $V = X/Y$
67.  $f(t) = n(n - 1)\lambda[\exp(-(n - 1)\lambda t) - \exp(-n\lambda t)]$