STATISTICS OF EARTHQUAKES

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INTRODUCTION

Earthquake statistics are facts recorded in the aftermath of seismic events. They are the concern of seismologists, geologists, engineers, government officials, insurers, and statisticians among others. Earthquakes provide special opportunties to learn about the makeup of the solid Earth. In the parlance of system identification, an earthquake is a pulse input at the event's origin to the system (Earth) having as responses seismograms observed around the Earth. A seismogram is a recorded time series of the displacements, velocities, or accelerations experienced by a particle at a location of the Earth. Figure 1 presents an example. It is a part of a record of the Earth's vertical motion as observed at Uppsala, Sweden, on April 20, 1989. It is highlighted here because of its use, together with a physical model and statistical methods, to learn about the surface composition of the Earth between Siberia and Upsalla (Bolt and Brillinger [12]). From seismograms recorded around Europe, using nonlinear regression analysis seismologists inferred that this event originated in the Sakha Republic of Russia. In the record one notes a variety of wiggles and fluctuations of varying amplitudes and periods. Seismologists attach physical significance to such features recording specific values such as arrival times of waves of differing types and routes through the Earth. The oscillations around the 30-second mark in the figure correspond to a Rayleigh wave train. Data concerning great earthquakes have been noted in China for more than two thousand years (Bolt [10]; Gu [25]). In particular there was a major collection of data immediately after the great Lisbon earthquake of 1755. These data are proving of high importance these days, as the papers in Mendes-Victor [47] show.

Statistics and statisticians are involved because of the large amount of and many forms of data that become available following an earthquake as well as the related scientific and social questions arising. Statistical methods have played an important role in seismology for many years in part because of the pathbreaking efforts of Harold Jeffreys (see Bolt [9]). Concerning Jeffreys' work, Hudson [30] has written: "The success of the Jeffreys–Bullen travel time tables was due in large part to Jeffreys' consistent use of sound statistical methods." In particular, Jeffreys' methods were robust and resistant, i.e., dealt with nongaussian distributions and outliers. Bolt [8] extended them to the linear regression case.

Statistics enters for a variety of reasons. For example, the basic quantity of concern may be a probability model or a risk. Further, the data sets are often massive and of many types. Also there is a substantial inherent variability and measurement error. In response, these days seismologists and seismic engineers continually set down stochastic models. Consider, for example, the Next Generation of Attenuation (NGA) (Stewart [64]). Such models need to be fitted, assessed, and revised. Inverse problems with the basic parameters defined indirectly need to be solved (O'Sullivan [53]; Stark [63]). Experiments need to be designed. In many cases researchers employ simulations and massive databases of such have been developed (see Olsen and Ely [52]). It can be noted that new statistical techniques often find immediate application in seismology particularly and in geophysics generally. In parallel, problems arising in seismology and earthquake engineering have led to the development of new statistical techniques.

Seismology underwent the "digital revolution" in the nineteen-fifties and continually poses problems exceeding the capabilities of the day's computers. Its participants have turned up a variety of empirical laws (Kanamori [37]). These prove useful for extrapolation to situations with few data (e.g., Huyse [32], Zhuang [78], and Amorèse [3]). Physical theories find important application (Aki and Richards [1]). The subject matter developed leads to hazard estimation (Wesnowski [76]); improved seismic (Naeim [48]; Mendes-Victor [47]); earthquake prediction (Zechar [77]; Lomnitz [42]; Harte and Vere-Jones [28]; Luen and Stark [43]); determination

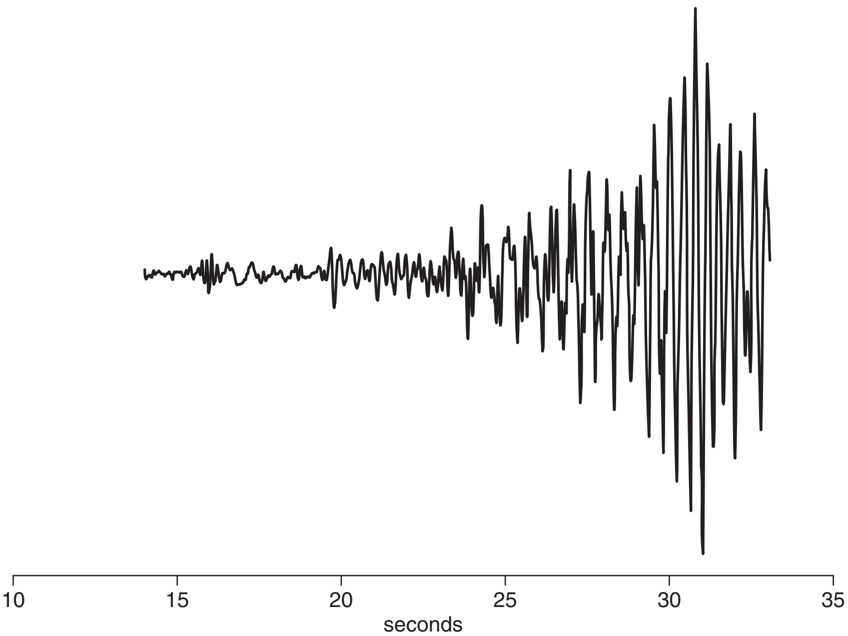

Figure 1. The vertical motion observed at Uppsala, Sweden of the of April 20, 1989 event originating in Southwestern Sakha, Russia.

of insurance premiums (Brillinger [16]; Kunreuter and Roth [40]); general knowledge of the structure of the Earth (Bolt [10]).

The field of seismology was almost totally observational for many years, but nowadays experiments have become common—for example, sending a seismometer to the moon, launching sensors in satellites, inputting impulses to the Earth in the search for gas and oil and learning its layers, and setting out sensors in specific designs. Analyses of the resultant data make continual use of statistical ideas and methods.

Other problems addressed by earthquake researchers and statisticians include: the detection, location and quantification of seismic events; risk assessment (Cornell [19]); prediction of earthquakes [42]; and the distinguishing of earthquakes from nuclear explosions (Bonner et al. [13]). The researchers are interested in structural questions such as: How should seismometers be laid out in a network? (Uhrhammer [68]); How well can one predict earthquakes? (Lomnitz [42]; Rhoades [56]; Zechar et al. [77]); Are layer boundaries flat or bumpy? (Pulliam and Stark [54]); and Is activity on different faults associated? Further, in problems of risk assessment, there is a need for attenuation laws providing the falloff of the strength of an earthquake's effect with distance from the seismic source (Stewart [64]). Algorithms are needed for automatically detecting the onset of a strong earthquake and in consequence then shutting down a critical facility (Allen [2]). Statistical ideas and methodology contributes to the solution of each of these problems. General references providing basic seismological background include: Aki and Richards [1], and Bullen and Bolt [17].

SOME TYPES OF DATA

Measurements may be made close to the source of an earthquake or at a distance, which may be great. Different equipment and models may be employed in these cases. The recording of data may be continual, as at an observatory, or brief, as when strong-motion instruments are triggered by substantial motion. The data processed may be the seismogram $Y(t)$ observed at equispaced times $t = 0, \ldots, T-1$. This is the case Figure 1. Complex derived quantities may be based upon such a record. An example is provided by Figure 1 in Bolt and Brillinger [12]. It provides a dynamic spectrum, which may be employed to infer the presence of modes and to estimate Earth densities and velocities. Other derived values include: first arrival time, direction of first motion (Udias [67]), amplitude of a particular first-motion, signal duration, maximum overall amplitude, and oscillation periods. Characteristic features may be noted to infer the individual arrival times of superposed waves of different types (Simon [62]). The time series recorded by a modern seismometer at a location are typically trivariate (two horizontal and one vertical components recorded), and may then be denoted $\{(Y_1(t), Y_2(t), Y_3(t)),$ $t = 0, \ldots, T-1\}$. There may be arrays of seismometers all of whose seismograms are employed jointly. An earthquake signal may thus be seen to be traveling and changing shape in its movement across the array. Data recorded at latitude–longitude (x_j, y_j) may be denoted $Y_j(t)$, $t = 0, \ldots, T-1$, with j labeling locations. See Roult [57], Shumway [60], and Thomson [65] for examples.

Derived values such as an event's origin time, location, and magnitude—$\{\tau_k, (x_k, y_k, z_k),$ $M_k,\ k = 1, 2, \ldots, K\}$, with k indexing events—may be collected into catalogs for geographic regions of interest. See Chiou et al. [18], the *Advanced National Seismic System Composite: A world-wide catalog* ([4]), and the *Chinese Historical Catalog* (Gu [25]). See also Veneziano [69], Schoenberg and Bolt [58], and Vere-Jones [71]. These catalogs can date back centuries and are fundamental tools of modern seismological research.

Sometimes seismological data are not based directly on seismograms. For example, they may be subjective assessments of damage following an event (Bullen and Bolt [17]; Bolt [10]). The modified Mercalli intensities are used for such a description. They are ordinal-valued. The description of MM intensity VI starts:

> *Felt by all; many frightened and run outdoors. Some heavy furniture moved . . .*

while that of intensity VII starts:

> *Everybody runs outdoors. Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable in poorly built . . ..*

Other types of data are collected to address other questions: decay of maximum acceleration with distance (Bolt and Abrahamson [11]); motion of tectonic plates (Bird [7]); asking if there are bumps on a layer (Pulliam and Stark [54]); creating "shake maps" from caller data (Wald [74]); creating isoseismal maps from damage reports (Brillinger [16]); and employing twitter reports (Earle et al. [22]).

MODELS

Models are mathematical idealizations of reality. They range from the naive—like the exponential distribution for magnitudes (the so-called Gutenberg–Richter relation) [26]—to the massive and sophisticated (Bebbington et al. [6]). Those commonly employed include binomial, Gaussian, Poisson, complex spatial–temporal, and branching. Seismic engineers proceed by developing stochastic models for the response of a building to seismic input, while seismologists may model the Earth's interior as random to handle the omnipresent irregularities (Hudson [31]).

An exceedingly broad range of stochastic models have been employed by researchers. These can provide effective summarization of the data and allow the addressing of questions of interest. For example, the sequence of times $\{\tau_k\}$ of earthquake occurrence in a given region may be viewed as corresponding to part of a realization of a stochastic point process. It becomes a marked point process or jump process $\{(\tau_k, M_k)\}$ when there is a value (mark) associated with each event time. This could be the event's magnitude or seismic moment. A basic point-process parameter, the rate, tells how many earthquakes may be expected in a unit time interval. There are other parameters to describe temporal dependence. A random field or spatial process, $Y(x, y)$, can be envisaged as describing realized values of (say) maximum displacements occurring at locations (x, y) on the Earth's surface during the course of an earthquake. A fluctuating displacement value in time and space, $Y(x, y, t)$, may be viewed as a spatial−temporal process. A branching process may correspond to crack or geological-fault formation or underlie the times and locations of events (Vere-Jones [72]).

There are many uses made of the models of elementary statistics such as multiple regression and nonlinear regression. The generalized linear model is being employed for data that are counts or proportions or that are necessarily positive.

STATISTICAL METHODS

At the outset of a discussion of statistical techniques for earthquake analysis one can mention methods based on: averaging, smoothing, least squares, moments, likelihood, random effects, robust/resistant procedures with their variants for convolved data, and the frequency domain. There have been some reviews of statistical techniques applied to earthquake data. Jeffreys [34] describes the methods employed through the mid-sixties. Vere-Jones and Smith [73] provide a review of many contemporary instances up through 1980. There have been applications of dimension estimation techniques (Scherbaum et al. [58]). Other recent references include Walden and Guttorp [75] and Vere-Jones [72]. See also Stark [63], who provides an incisive review of statistics in geophysics.

Part of modern seismological research is based on the spectral analysis of seismograms; see Bath [5]. Many specific statistical methods that have been employed involve maximizing a likelihood function (e.g., Guttorp and Hopkins[27]); including measurement error (Ganse et al. [23]); robust regression variants (Bolt [8]); nonlinear regression (Bolt and Abrahamson [11]); probit analysis (Vere-Jones and Smith [73]); Fourier inference (Ihaka [33]); discrimination (Tjostheim [66]); array analysis (Shumway [61]; Thomson[65]); point processes (Vere-Jones [70]; Cornell [20]; Hawkes [20]; Kirmedjian and Suzuki [38]; McGuire [45]; Venezeano [69]); moment functions (Kagan [35]); inverse problems (O'Sullivan [53]); bootstrap (Lamarre [41]); and sensitivity analysis (Rabinowitz [55]).

The smoothness-priors approach to nonstationary data (Kitagawa and Gersch [39]) leads to dynamic spectra, plausible plots of time-varying frequency content of seismic signals. Mendel [46] presents maximum-likelihood state-based methods for handling the data of reflection seismology, while Der et al. [21] indicate how the EM method may be employed to deconvolve pulses confounded in seismic traces. The non-Gaussianity of seismograms is taken advantage of in higher-order moment analysis (Gianakis [24]). Researchers Ogata [49−51], and Kagan [36] have carried out a variety of likelihood-based analyses of earthquake times as a point process.

An important conceptual development is the systems approach of employing box-and-arrow diagrams to break down a circumstance into simpler components for modeling. This is the case of problems of seismic risk analysis (Cornell [19]). Brillinger [14] and [15] present a variety of statistical analyses of earthquake data.

THE LITERATURE

The principal journals in the field of earthquake statistics include *Bulletin of the Seismological Society of America, Journal of Geophysical Research, Geophysical Journal of the Royal Astronomical Society, Geophysical Research Letters*, and *Mathematical Geology*.

The field of seismology has always been remarkable for the speed with which the data are shared. Nowadays catalogs and waveforms may be obtained directly from many observations through the Internet; see Malone [44]. One list of computer addresses is given in Bolt [10].

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