

A meandering *hylje**

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Abstract. Hawaiian monk seals (*Monachus schauinslandi*) are endemic to the Hawaiian Islands. The species has been declining for several decades and now numbers around 1300. A key hypothesis accounting for the decline is poor growth and survival of young seals owing to poor foraging success. Consequently, data have been collected recently on the foraging habitats, movements, and behaviors of Hawaiian monk seals throughout the Hawaiian Islands Archipelago.

Our work here is directed to exploring a data set located on the west side of the main Hawaiian Island of Molokai in our search for a stochastic model of a seal's journey. The work proceeds by fitting a stochastic differential equation (SDE) that mimics some aspects of the behavior of seals by working with location data collected for one seal. The SDE is found by developing a potential function. The estimated times of locations are irregularly spaced and not close together leading to some difficulties of analysis and interpretation. Synthetic plots are generated to assess the reasonableness of the model and suggest departures from it.

2000 MSC codes: 60J60, 62G08, 62M10, 70F99.

Key words and phrases: ARGOS satellite locations; Hawaiian monk seal; Potential function; Spatial locations; Stochastic differential equation; Synthetic plot.

1 Introduction

Tarmo Pukkila has made many contributions to statistics in general and to time series specifically. Figure 1 below shows a particular bivariate time series that will be studied in this paper. We would like to be able to apply some of Tarmo's work on ARIMAs to the analysis of this series, but the series is seen to be plagued by various complications including: outliers, unequally spaced time sampling intervals and does not look like any of the usual ARIMAs. What gives some hope in developing an analysis, as we shall see, is that a

*Finnish for seal.

great deal is known about the series context. However surely at later stages Tarmo's work will be applicable and probably applied.

Populations of virtually all pinnipeds (i.e., seals and sea lions) were reduced substantially, and some nearly extinguished, during the 1800s and 1900s by commercial sealers and whalers, poachers, and human fishers who have considered these predators to be competitors for shared marine resources (Reeves et al. 1992; Reeves and Stewart 2005). Most species however, have recovered from those residual small populations and are now relatively robust. There are exceptions however. The Saimaa seal (*Pusa hispida saimensis*), the Ladoga seal (*Pusa hispida ladogensis*), and the Mediterranean (*Monachus monachus*) and Hawaiian (*Monachus schauinslandi*) monk seals are the clearest cases of populations or species that are still at substantial risk of extinction from various causes (e.g., stochastic fluctuation in abundance, environmental change, habitat modification or destruction, reduction in prey, anthropogenic mortality of seals). The Saimaa and Ladoga seals are subspecies of the parent ringed seal species that is widely distributed in the circumpolar Arctic. Both of those species have been confined to inland freshwater lakes for about 9,000 to 10,000 years, the former to the Saimaa lake system in eastern Finland and the latter to Lake Ladoga in Russia near St. Petersburg (Reeves et al. 1992, 2002). Though perhaps never historically exceeding 2000 to 2500 seals, the Saimaa seal declined to around 200 seals by the 1980s, though has evidently increased some recently to around 250 to 300 owing to direct intervention and conservation measures.

Though more numerous and with a less confined distribution than the Saimaa seal, the Hawaiian monk seal now numbers only around 1300 seals in the Hawaiian Island Archipelago, having declined substantially from the 1950s through at least the late 1990s. Because the population is predominately older seals, it is predicted to decline further during the next two decades at least as the number of recruits to the breeding population will be small. This bias towards a mature and aging population is related to poor growth and survival of young seals, evidently owing in part to their poor foraging success, (Craig and Ragen 1999; Baker and Johanos 2004; Stewart et al. 2006). Consequently, data have been collected recently on the foraging habitats, movements, and behaviors of monk seals throughout the Northwestern and main Hawaiian Islands.

This paper studies a three month journey of a juvenile male Hawaiian monk seal, while he foraged and occasionally hauled out ashore. The track started 13 April 04 and ended 27 July 04. He was tagged and released at the southwest corner of Molokai, see Figure 2. He had a satellite-linked radio transmitter glued to his dorsal pelage to document geographic and vertical movements as proxies of foraging behavior. There were 754 locations estimated in all, but many were suspicious. Understanding the foraging behavior and habitat use of the Hawaiian monk seal is critical for clearly identifying the causes for it and instituting management responses to end and reverse

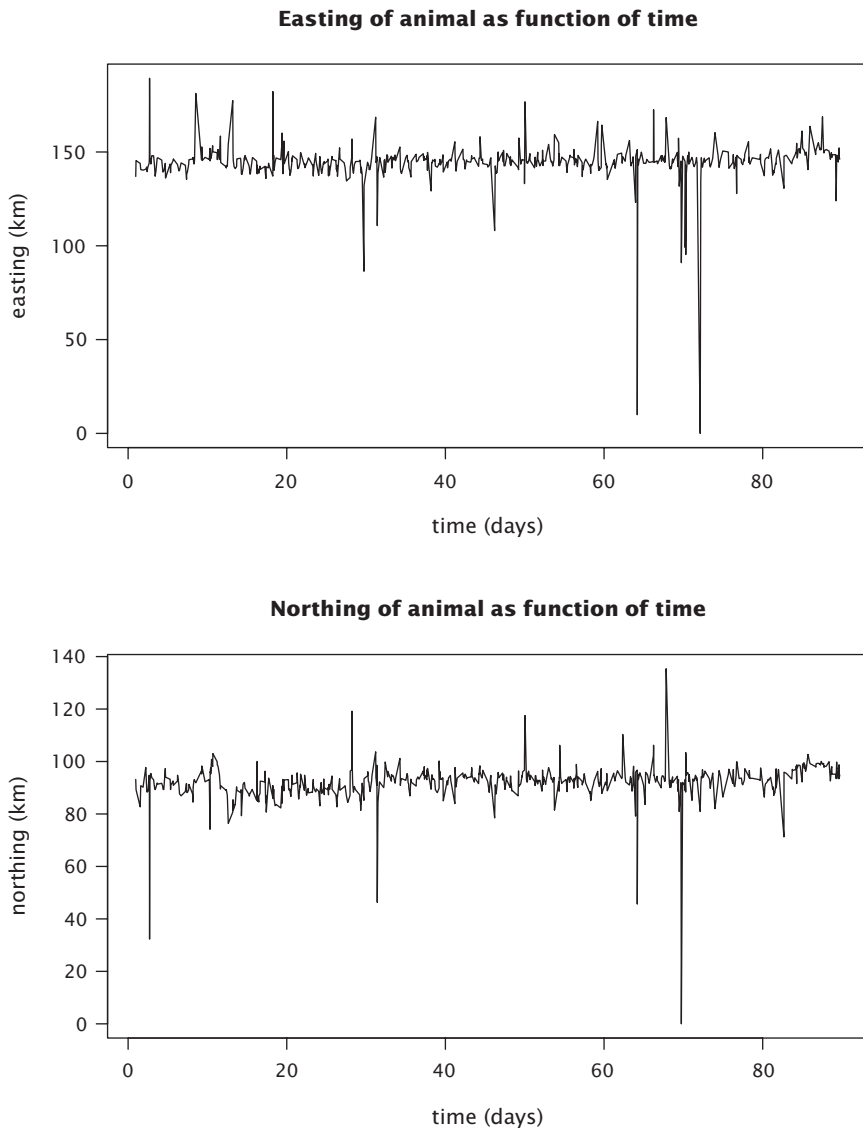


Figure 1. The top graph provides the eastward movement of the seal as a function of time and the bottom the northward component of his movement.

it. There are various specific questions that are relevant to this end:

- “What are the geographic and vertical marine habitats that the Hawaiian monk seals use?”
- “Are there age and sex differences in the habitats seals use when foraging?”
- “Do seals have individual preferences in foraging locations and does an individual vary its behavior over different time scales?”
- “How long is a foraging trip?”

This paper will focus on the first and fourth questions.

The paper Brillinger et al. (2006) presents an exploratory statistical study of the movements of a 4 year old female seal released at the same time and location as the seal of this paper. In those analyses we found that the animal made separate journeys to nearby Penguin Bank. The original intent of our current analyses was to confirm the findings of the first paper using data from another seal. Though we did not confirm those details, we did find that the same general approach of using a potential function proved viable.

A basic tool of the work is a stochastic differential equation. The SDE approach is elaborated in Brillinger et al. (2002). The potential function approach may be found in Brillinger et al. (2001b). There are many references to the literature in those papers. Two particular are Heyde (1994) and Sorensen (1997).

As in the previous paper, we found several difficulties with the data including: outliers, well-separated unequally spaced observations times, map creation, changing coordinates, developing the potential function, observation error (points inland and unreasonable speeds) and carrying out the simulations.

The paper includes the sections: The data and some initial analyses, Gradient functions, Fitting an SDE, Results, Validation, and Discussion.

2 The data and some initial analyses

The study began with a satellite-linked time depth recorder (SLTDR) being glued onto the seal’s dorsal pelage. The SLTDR records times and depths of dives and transmits a brief radio frequency signal to a system of near polar earth orbiting satellites managed by the Argos Data Collection and Location Service. Periodic locations of the seal being studied were determined by measurements of Doppler shifts in the reception of successive transmissions. Those locations and accompanying dive data were then communicated daily to us by email.

Associated with a location estimate is a prediction of the location’s error (LC or location class). The LC index takes on the values 3, 2, 1, 0, A, B, Z. When LC = 3, 2, or 1 the error in the location is predicted to be 1 km or less.

Errors for locations of $LC = 0$, A, and B are not predicted by Argos but may often be around 1 km but up to 10 to 20 km on occasion. In this paper when LC is 3, 2, or 1, a location is referred to as well-determined.

Figure 2 (bottom plot) provides locations with LC of value less than 1 and in the top plot those with LC 1 and up. (Later there will be a further restriction that the speed between any two successive location points is less than 259.2 km/day.) The figure is meant to provide an indication of the distinction between well- and poorly-determined points, (i.e. of the impact of the LC value.) One sees that the well-determined points are clustered close to the SW corner of Molokai while the poor ones are some distance offshore and in one case far offshore.

In the work it is convenient to employ UTM coordinates instead of the traditional latitude and longitude. This reduces distortion in the plots, leads to units of km, allows easy interpolation, and generally speaking makes the results more easily interpretable.

Figure 3 provides plots of the well-determined points for 6 successive 15 day periods. The dashed curve is the 200 fathom line which outlines Penguin Bank to some extent. (Penguin Bank is a marine reserve that is attractive to sea life.) The time points are unequally spaced. This means that apparent hotspots may well result from a cluster of time locations rather than a tendency of the animal to return to some particular location.

Figure 4 provides an estimate of the density of the well-determined points as a function of location. The lighter colors refer to the large values. One sees a "hotspot" partly up the coast and the points generally lying in a strip running NE-SW. The hotspot may be simply due to a lot of points being close to each other in time.

The behaviors of many animals are often characterized by circadian (i.e., 24-hour) rhythms. Such behavior was apparent in the movements of elk, see Brillinger et al. (2001a,b, 2002), Preisler et al. (2004), though it was not noticeable in our previous study of a female Hawaiian monk seal, Brillinger et al. (2006).

Figure 5 graphs estimated speeds of the animal based on the well-determined points. A log scale is employed to make the fluctuations more nearly constant. A smooth line has been added to the plot, specifically Cleveland's loess line, see Venables and Ripley (2002). There is a clear suggestion of the animal's moving more rapidly between 0500 and 1000 hr. The speed of the animal at the observation times was estimated by dividing the distance between successive points by the time distance. It is to be noted that there are some gaps in hours of the data. These are caused by the particular orbits of the satellites and complicate the interpretation.

Figure 6 is a plot of successive distances of the animal from the closest point on the coast. In Brillinger et al. (2006) such a plot brought out a number of offshore trips. In the present case one sees that the animal does spend some noticeable time offshore.

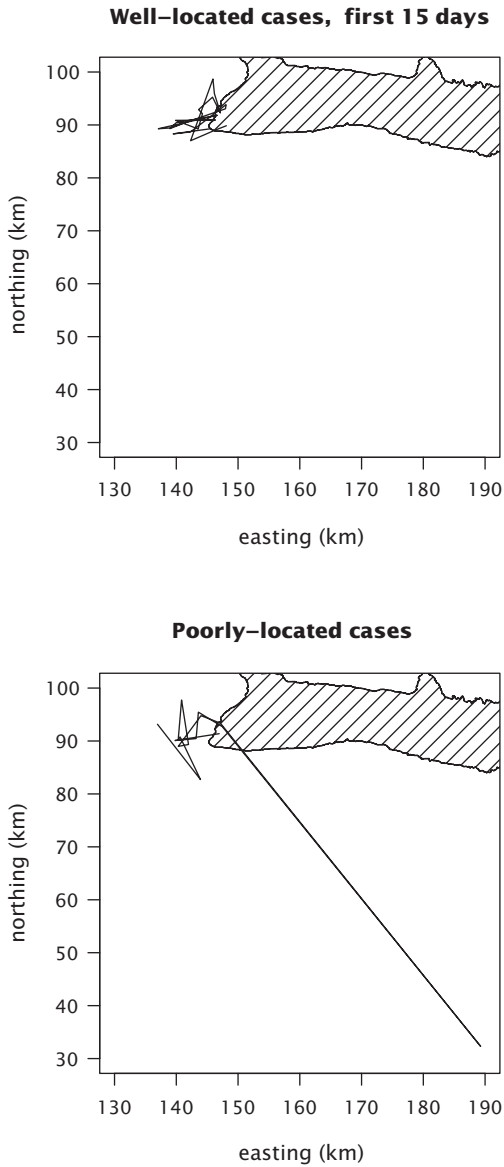


Figure 2. The top figure shows the well-determined points joined by successive lines. The bottom provides the poorly determined points. The hatched in area indicates the island of Molokai. The animal’s tracking started at the SW corner of the island.

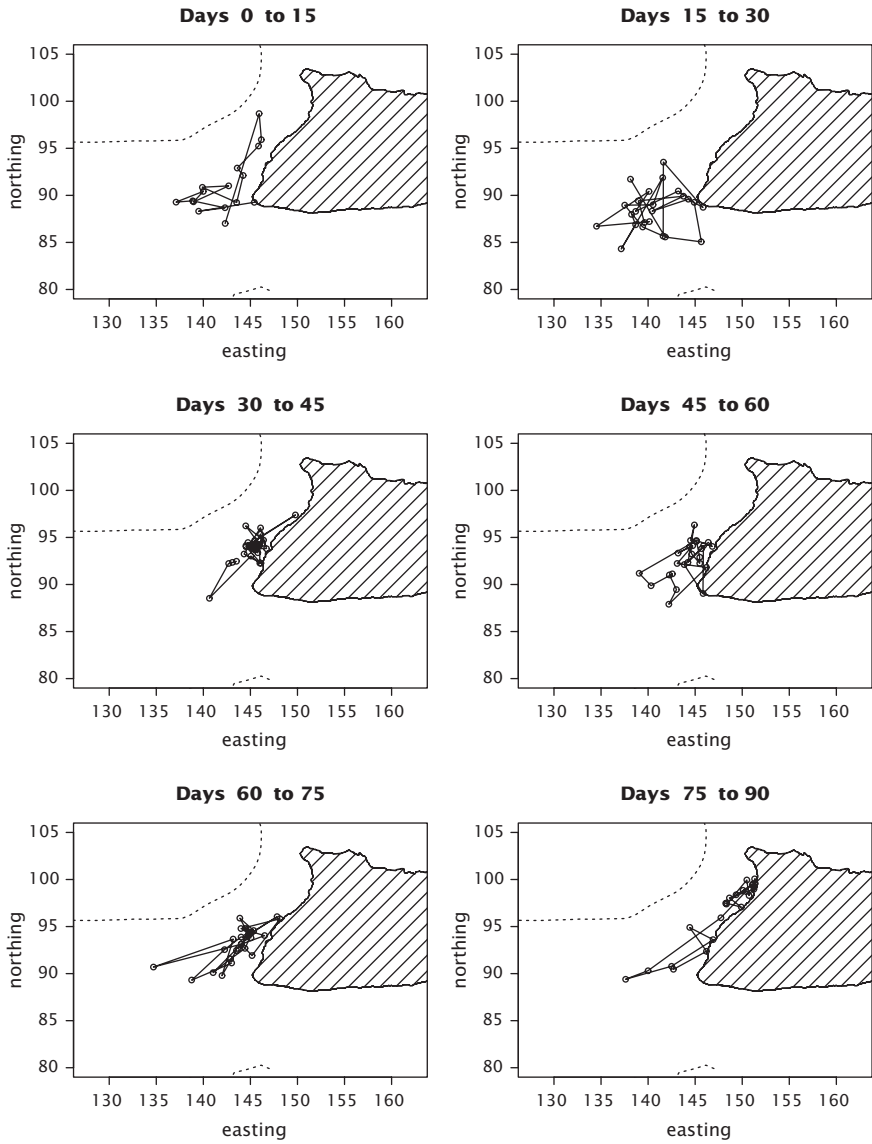


Figure 3. Plots of the well-determined points for six successive 15 day periods. The dashed curve is the 200 fathom line.

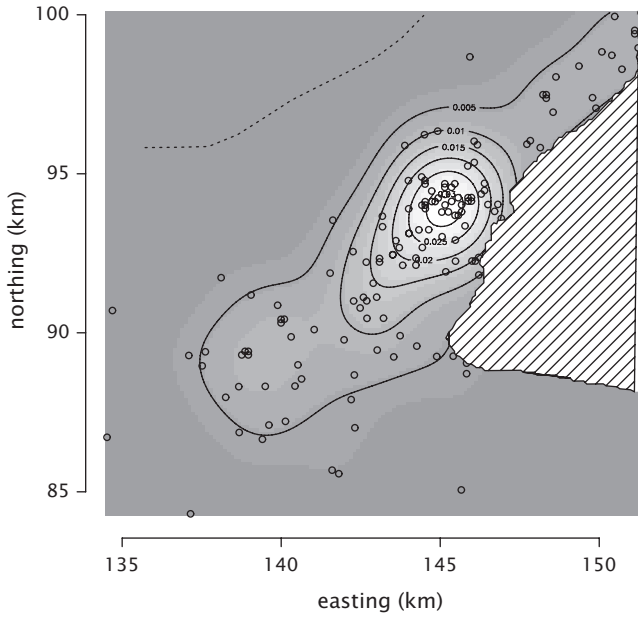


Figure 4. Estimated bivariate density of observed seal locations. Molokai is the hatched region. Lighter coloring corresponds to higher density.

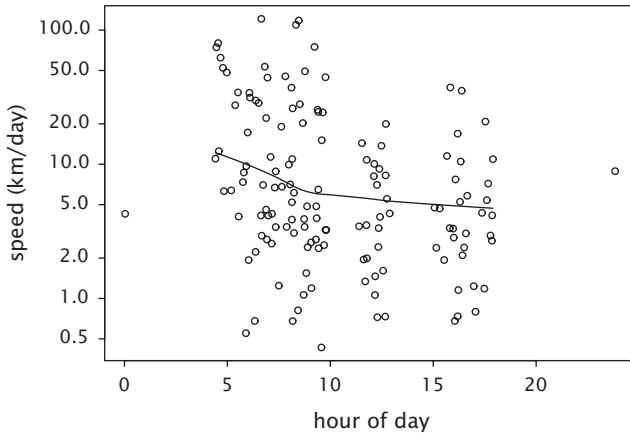


Figure 5. Log-scale plot of estimates of the animal's speed plotted versus time. A loess line has been added. The early morning and late night gaps are due to the satellite's orbit

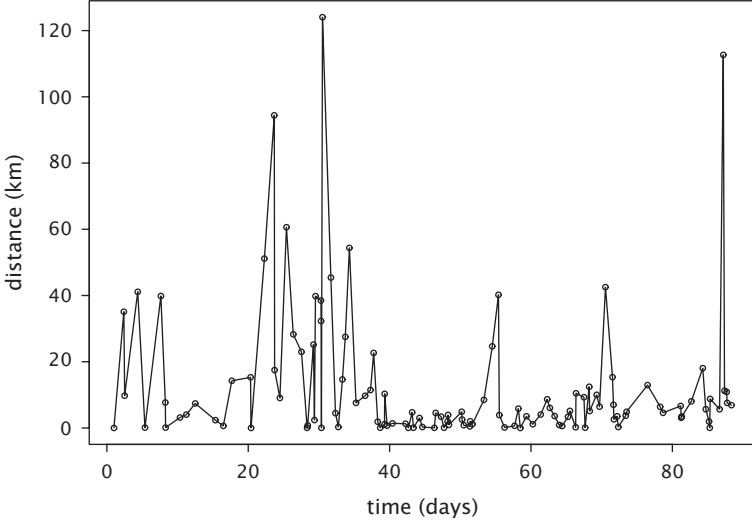


Figure 6. Successive distances of the seal from the shore.

3 Gradient functions

The study of motion, and the statistics of motion, has a long and venerable history. To begin one can note the Newtonian equations of motion

$$d\mathbf{r}(t) = \mathbf{v}(t) dt$$

$$d\mathbf{v}(t) = -\beta\mathbf{v}(t) - \beta\nabla H(\mathbf{r}(t), t) dt$$

with $\mathbf{r}(t)$ a particle's location at time t , with $\mathbf{v}(t)$ its velocity, with $H(\mathbf{r}, t)$ a potential function, and with ∇ the gradient. The potential function controls a particle's direction and velocity. Regions of attraction and repulsion may be introduced by terms in H . The parameter β represents friction. Nelson (1967) is a reference for this material.

In the case that β is large, the equations become, approximately,

$$d\mathbf{r}(t) = -\nabla H(\mathbf{r}(t), t) dt$$

If one adds a stochastic term, and changes the notation slightly, then one obtains the stochastic differential equation,

$$d\mathbf{r}(t) = \boldsymbol{\mu}(\mathbf{r}(t), t) dt + \boldsymbol{\Sigma}(\mathbf{r}(t), t) dB(t) \quad (1)$$

Often $B(t)$ is assumed to be standard Brownian motion. In the planar case with $\mathbf{r} = (x, y)$ the 2-vector $\boldsymbol{\mu}$ contains the partial derivatives H_x, H_y .

The following potential function was suggested for the seal of this paper by Figures 3 and 4,

$$H(x, y) = \beta_{10}x + \beta_{01}y + \beta_{20}x^2 + \beta_{11}xy + \beta_{02}y^2 + C/d_M(x, y) \quad (2)$$

where $d_M(x, y)$ is the shortest distance from the location (x, y) to Molokai. (In the results presented $C = 7.5$ a value suggested by a few trials.) Terms like $C/d_M(x, y)$ are considered in Brillinger et al. (2001b). It has the effect of keeping the animal away from the interior of Molokai. In the computation of Figure 7 below the shortest distance +0.2 was taken for $d_M(x, y)$. The 0.2 was to stabilize the computations. The quadratic in (x, y) allows a broad region of attraction to be present.

4 Fitting an SDE

Stochastic differential equations, such as (1), have been referred to. There has been a substantial amount of work on statistical inference for SDEs, references including Heyde (1994) and Sorensen (1997). Inferential work may be motivated by setting down the Euler approximation

$$\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i) \approx \boldsymbol{\mu}(\mathbf{r}(t_i), t_i)(t_{i+1} - t_i) + \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i)\mathbf{Z}_i\sqrt{t_{i+1} - t_i} \quad (3)$$

with the t_i an increasing sequence of time points filling in the time domain of the problem, see Kloeden and Platen (1995). The \mathbf{Z}_i are independent bivariate standard normals and the t_i may be thought of as the times of observation. With the potential function set down above the β parameters appear linearly and so may be estimated by least squares.

Assuming that $\boldsymbol{\mu}(\mathbf{r}, t) = \boldsymbol{\mu}(\mathbf{r})$, and that $\boldsymbol{\Sigma}(\mathbf{r}(t), t) = \sigma^2\mathbf{I}$, one can consider as an estimate of σ^2

$$\hat{\sigma}^2 = \frac{1}{I} \sum_i \|\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i) - \hat{\boldsymbol{\mu}}(\mathbf{r}(t_i))(t_{i+1} - t_i)\|^2 / (t_{i+1} - t_i) \quad (4)$$

$i = 1, \dots, I$ having determined an estimate of $(\beta_{10}, \dots, \beta_{02})$ by least squares.

5 Results

The model is

$$d\mathbf{r}(t) = \boldsymbol{\mu}(\mathbf{r}(t)) dt + \sigma d\mathbf{B}(t), \quad \mathbf{r}(t) \in F \quad (5)$$

with F a region to be described, with the potential function (2) and with \mathbf{B} bivariate Brownian. The region F is the area between the 200 fathom line and Molokai.

The number of data points in the least squares analysis was 142. The parameter estimates obtained were $\hat{\boldsymbol{\beta}} = (93.53, 8.00, -.47, .47, -.41)$, and $\hat{\sigma} = 4.64$ km.

Figure 7 shows the estimated potential function of (3). The particle (seal) is pulled into the middle of the concentric contours, but the Brownian term pushes it about. The final term of (2) keeps the "animal" off Molokai.

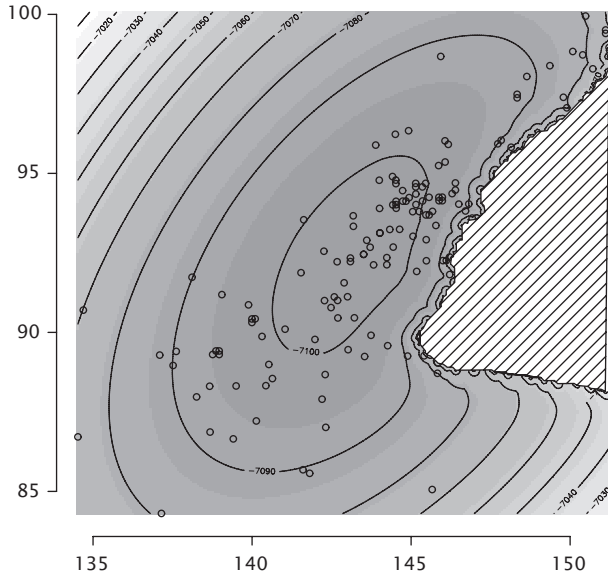


Figure 7. The fitted potential function obtained using the model (2). The darker values are deeper.

6 Validation

Figure 8 shows the results of a simulation of the process (only one was generated) taking the parameter values to be those estimated. The sampling interval $dt = t_{i+1} - t_i$ employed in the numerical integration is 1 hour. The paths were constrained to not go outside the 200 fathom line. The locations at the time points of the data set are the points plotted. This allows direct comparison with the data plot of Figure 3. The variability of Figure 8 is not unlike that of Figure 3.

The plots are “synthetic” in the language of Neyman et al. (1952), Neyman and Scott (1956). They are an exploratory tool for model validation having the possibility of suggesting how to create another model if the resemblance is not good.

Future work includes a study of measurement error, uncertainty, animal interactions and formal validation.

7 Discussion

The work searched out a stochastic model for a seal's trajectory using descriptive methods, classical dynamics and statistical techniques. Developing a pertinent potential function proved an effective manner by which to infer a model for the animal's track. There was preliminary model assessment by looking at pictures of simulations.

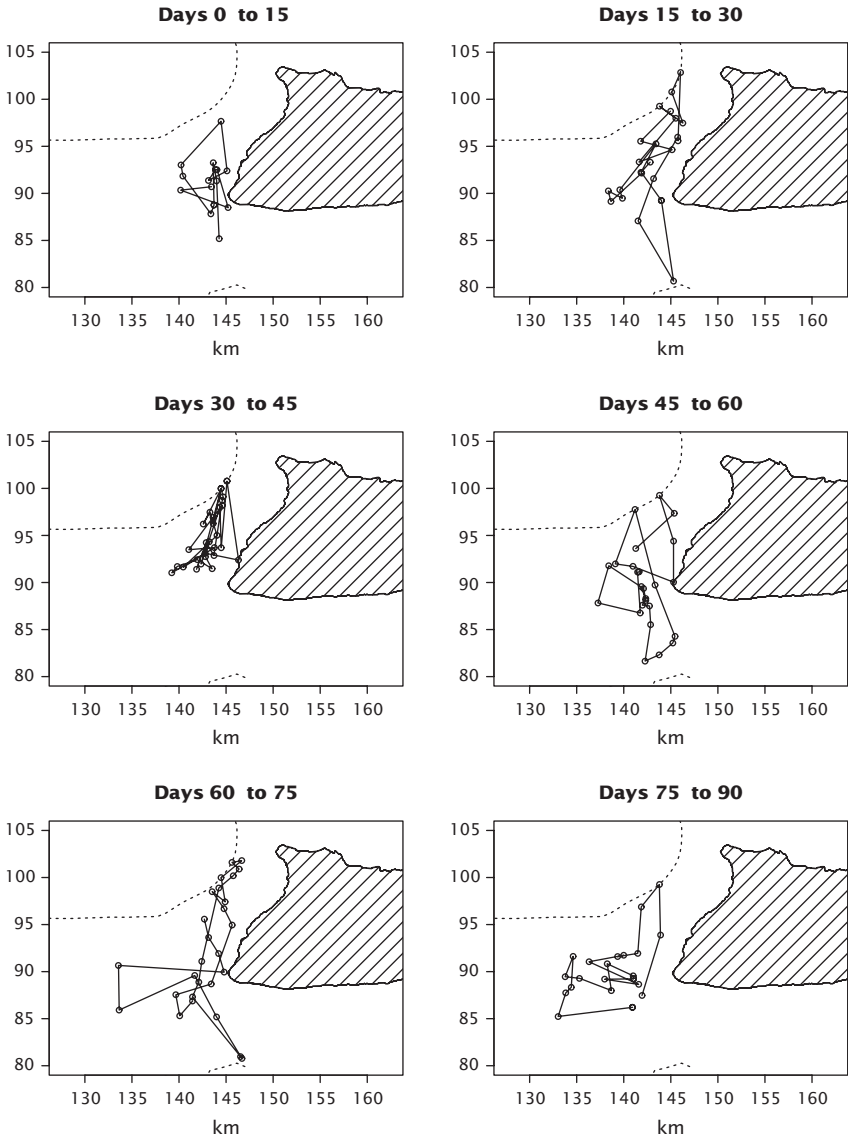


Figure 8. A simulation of the model (5) having fit the potential function (2). The times are those of the data of Figure 3.

It is to be noted that time of day was not included in model in contrast to the work of Brillinger et al. (2001a,b). The work is preliminary and circadian the effect was not considered overly strong. The work again constitutes an exploratory analysis, with the results to be examined with data to be had from other animals.

The key hypothesis accounting for the decline of Hawaiian monk seals in the primary part of their range, the Northwestern Hawaiian Islands, is that growth and survival of juvenile seals is poor owing to poor foraging success (Craig and Ragen 1999; Stewart et al. 2006). Seals in the small but growing population in the main Hawaiian Islands appear to be in better physical condition and grow faster and survive better than those in the Northwestern Hawaiian Islands. The results of the development of descriptive foraging movements of monk seals in the main Hawaiian Islands and of the simulations of movements that incorporate simple attractant and repellent elements may be helpful in further evaluating the differential dynamics. We think that those further studies, directed by these theoretical inquiries, will provide substantial insights into the reasons for the continued declines of monk seals in the Northwestern Hawaiian Islands versus the increases in the main Hawaiian Islands relative to habitat distribution and biological productivity.

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