Marine Mammal Movements: Data Analysis and Theory

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Brent Stewart, Hubbs

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Structure of presentation: 4 examples

Introduction

. “*The science of statistics is essentially a branch of Applied Mathematics, and may be regarded as mathematics applied to observational data.”*

R. A. Fisher (1925)

Scientific work is often motivated by concepts and methods from physics and mathematics

Pertient names: Newton, Einstein, Smoluchovsky, Langevin, Wiener, Chandresekhar, Nelson, Kendall, …

Beginning models abstract and mathematical but they motivate discrete time, programmable, checkable models.

Defence: providing answers to broad variety of questions.
e.g. interventions existing?, predicted locations, change?

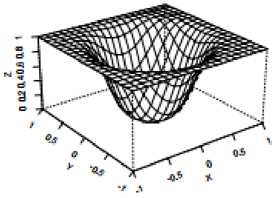
Goal is to estimate a potential function whose gradient appears in a (linear) model.
Then can use many regression results

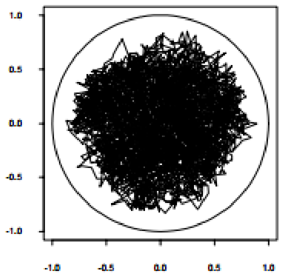
The mammals

 Northern elephant seal	 Monk seal
 Whale shark + tag Nor a mammal	 elk

Interpretation of a potential function

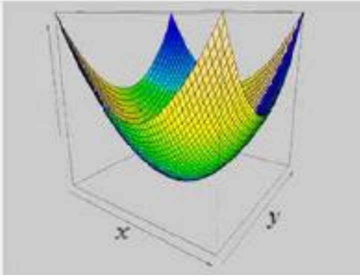
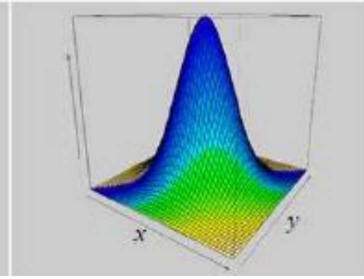
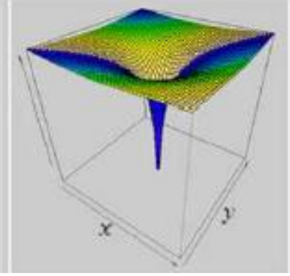
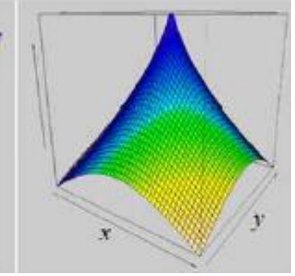
Motion described by SDE or gradient of a potential function

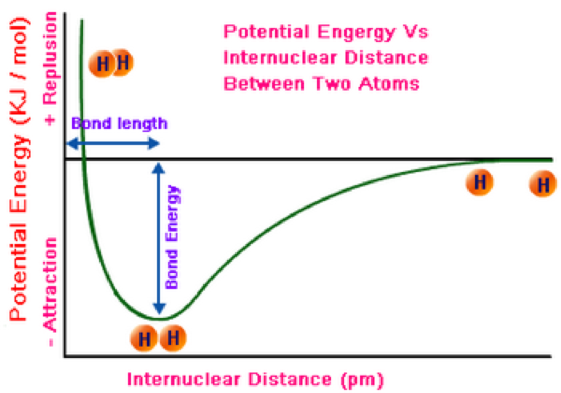




rolling ball bearing

types of potential functions: attraction, repulsion, time dependent, attractor-repellor, periodic, inverse power, polynomial, isotropic,

Example	Orenstein-Uhlenbeck	Gaussian	Gravitational	Zohdi
Potential function	$H=\beta_o+\beta_1(x-x_o)^2+\beta_2(y-y_o)^2$	$H=\exp\left\{\alpha(x-x_o)^2+\beta(y-y_o)^2\right\}$	$H=\beta/D$	$H=\alpha D^{-\beta}-\gamma D^{-\delta}$
Potential surface				



$$||r||^{\alpha}$$

$$\hat{U}(\mathbf{r})=\sum_j \hat{\alpha}_j \phi_j(\mathbf{r}),$$

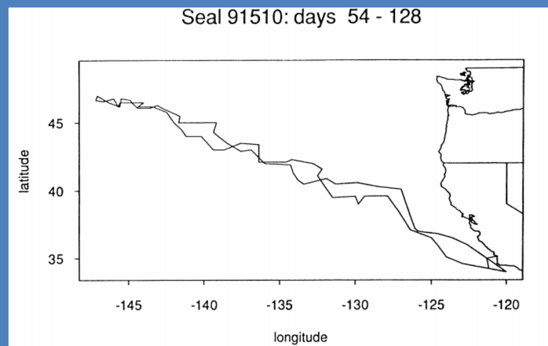
Path/track data from: elephant seals, monk seals, elk, whale shark tag

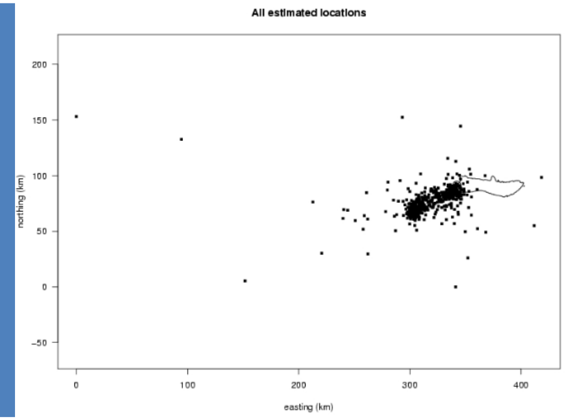
Reasons for study: endangered, coexistence possible?, discovery, management, prediction, change, outside influence?

Exploratory analytic method

Building on historical background and available statistical methods (EDA, least squares)

Data





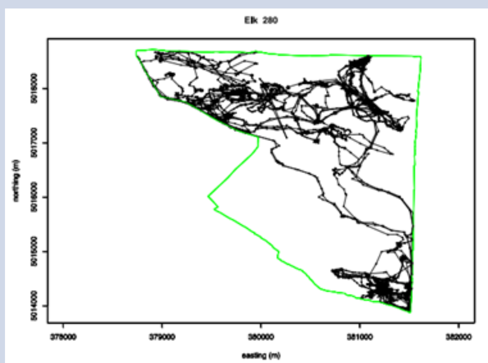
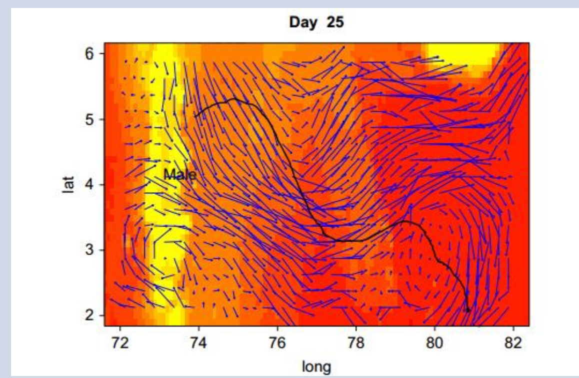


Figure 2 *Superposed paths of six animals for the first week.*

elk



Free floating tag

Stochastic calculus

Langevin-Chandresekhar equation

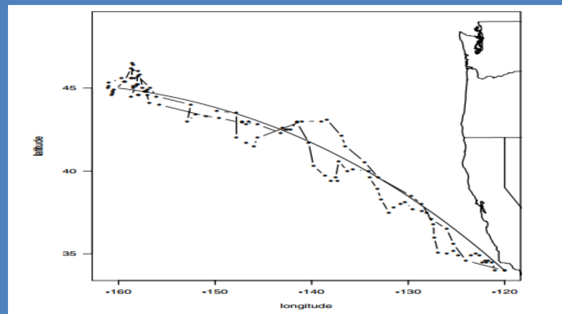
cp. F = ma

acceleration

$$\mathbf{v} = d\mathbf{R}/dt, \quad m d\mathbf{v}/dt = -\gamma\, \mathbf{v} + K(R) + \mathbf{\Psi}(t), \; \mathbf{K}(\mathbf{R}) = -\mathrm{grad}\ Б(\mathbf{r},t)$$

Assuming friction, γ, is high

velocity $d\mathbf{R}/dt \sim -\mathrm{grad}\ Б(\mathbf{r},t) + \mathbf{\Psi}(t)$



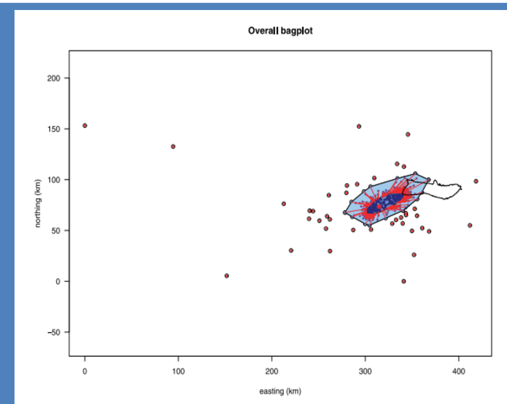
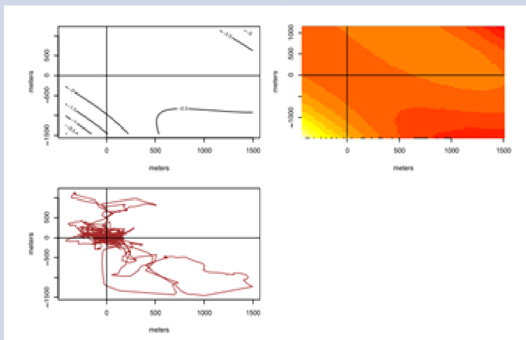
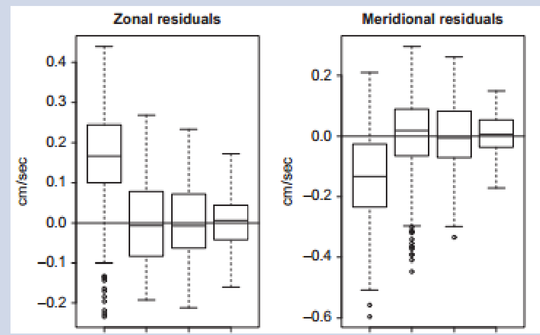


FIG 5. *Bagplot of all the estimated locations. The bag and fence add detail to Figure 2.*





Some results

Example 1. Northern elephant seal (*Mirounga angustirostris*)

Were virtually extinct

Size   male 2000 kg   female 600 kg

Exceptional navigators

Most of year at sea

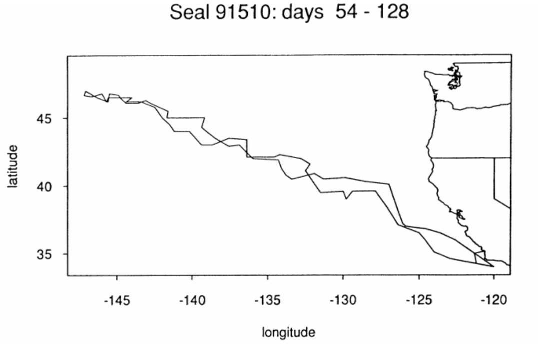
Double annual migrations

Able to assess position by astronmical or magnetic field and correct course???

forage continuously

EDA: discovery (visualization), need for robust/resistant methods

Great circl



Particle movement models

Model 1. patrcle model Random walk on sphere with drift, (lat, long) = (θ,φ)

D. G. Kendall model for birds, SDE

Change of variables so heading to North Pole

Equation of motion

$$d\theta_t = \frac{\sigma^2}{2\tan\theta_t}dt + \sigma\, dU_t$$

$$d\phi_t = \frac{\sigma}{\sin\theta_t}\, dV_t.$$

Potential function

$$H(\theta, \varphi) = -\tfrac{1}{2}\sigma^2 \log \sin\theta.$$

point of attraction North Pole

Inference

A discrete approximation to the model (8), (9) is provided by

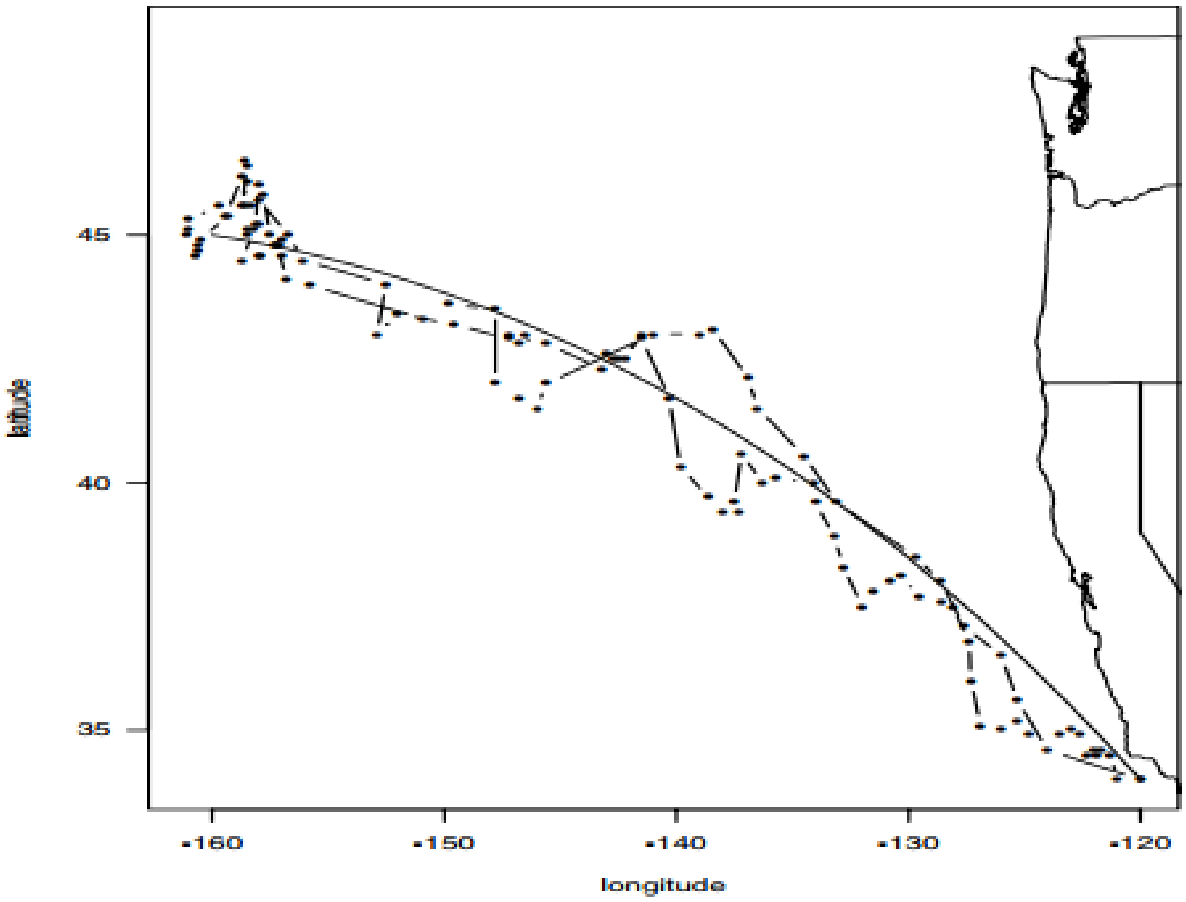
$$\theta_{t+1} - \theta_t = \frac{\sigma^2}{2 \tan \theta_t} - \delta + \sigma\epsilon_{t+1},$$

$$\phi_{t+1} - \phi_t = \frac{\sigma}{\sin \theta_t}\eta_{t+1},$$

-2 log like

$$2T \log \sigma^2 + \frac{1}{\sigma^2}\sum(\sin^2 \theta_t)(\phi_{t+1} - \phi_t)^2 + \frac{1}{\sigma^2}\sum\left(\theta_{t+1} - \theta_t + \delta - \frac{\sigma^2}{2 \tan \theta_t}\right)^2$$

Surprises . Brownian with trend on sphere, new model, path can be great circle



Discussion.

great-circle path hypothesis not contradicted

keep going straight ahead

one northern elephant seal female

Suggests seals can have a destination when departing from an origin

natural selection has favoured development of neural and sensory mechanisms

Example 2. Monk seal (*Monachus schauinslandi*

Endangered. Now numbers around 1100

Key factor in recent decline poor survival of juveniles
hypothesized related to poor foraging success

Basic motivation to learn where animals go to forage vertically and geographically.

Information needed for management and conservation purposes.

Example 2. Monk seal (*Monachus schauinslandi*

Endangered. Now numbers around 1100

Key factor in decline poor survival of juveniles
Hypothesized related to poor foraging success

Which geographic and vertical marine habitats seals use?

What habitats are essential, with some buffer, to the survival and vitality of this species?

Are there age and sex differences in habitats used when foraging?

Do seals have individual preferences in foraging locations and does an individual vary its behavior over different time scales?
how long is a foraging trip?

To begin: EDA   scatter plot of GIS positions

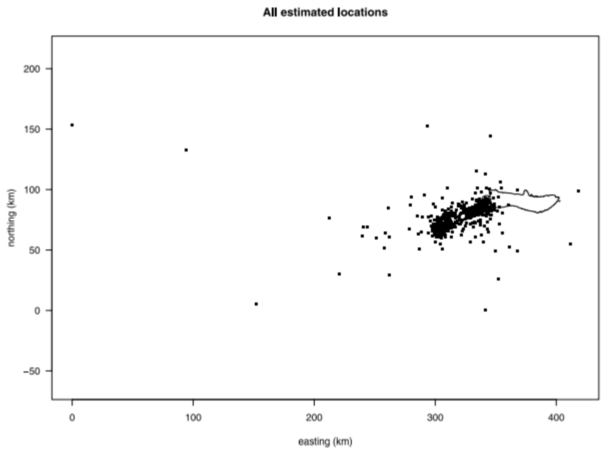


FIG 2. *The figure shows all 573 of the estimated locations. The island is Molokai. The circle indicates the initial estimated position. The coordinates are UTM.*

EDA bagplot (bivariate boxplot)

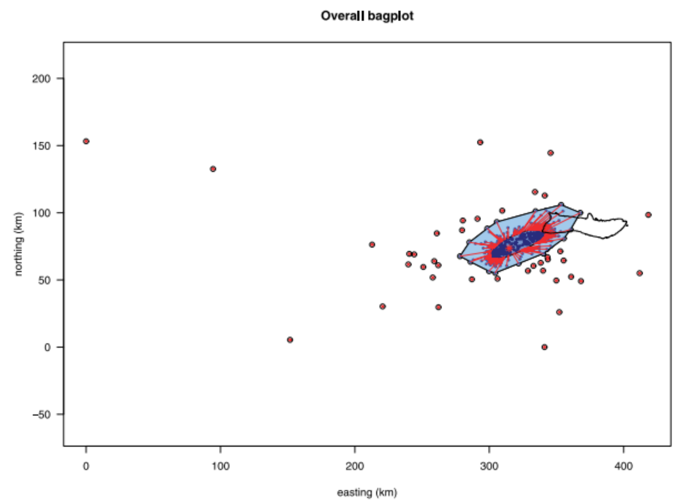


FIG 5. *Bagplot of all the estimated locations. The bag and fence add detail to Figure 2.*

Surprise: *Penguin Bank*

Model 2

$$d\mathbf{r}_t = -\ \text{grad}\ U(\mathbf{r}_t)dt + \mathbf{\Sigma}\ d\mathbf{B}_t$$

The potential function form employed in the computations to be presented is

$$\beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 xy + \beta_5 y^2 + \beta_6 x^3 + \beta_7 x^2 y + \beta_8 xy^2 + \beta_9 y^3$$

with (x, y) denoting location. The gradient is

$$\begin{pmatrix} 1 & 0 & 2x & y & 0 & 3x^2 & 2xy & y^2 & 0 \\ 0 & 1 & 0 & x & 2y & 0 & x^2 & 2xy & 4y^2 \end{pmatrix}$$

matrix multiplied by the transpose of the row vector

$$(\beta_1 \quad \beta_2 \quad \beta_3 \quad \beta_4 \quad \beta_5 \quad \beta_6 \quad \beta_7 \quad \beta_8 \quad \beta_9).$$

The result is linear in the β's. Because of this linearity simple multiple regression may be employed to obtain estimates. The steps of the analysis were described in Sections 2 and 3.

$$\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i) \approx \boldsymbol{\mu}(\mathbf{r}(t_i), t_i)(t_{i+1} - t_i) + \mathbf{\Sigma}(\mathbf{r}(t_i), t_i)\mathbf{Z}_i\sqrt{t_{i+1} - t_i}$$

Foraging trips

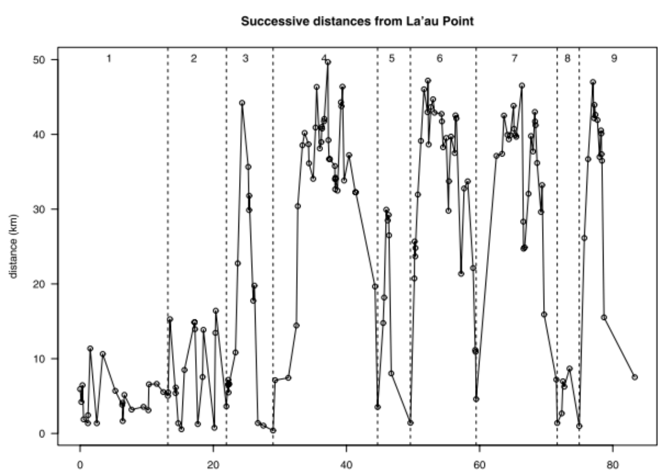
$$\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i) \;\approx\; \boldsymbol{\mu}(\mathbf{r}(t_i), t_i)(t_{i+1} - t_i) \;+\; \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i)\mathbf{Z}_i\sqrt{t_{i+1} - t_i}$$

$\boldsymbol{\Sigma}(\mathbf{r}(t), t) \;=\; \sigma\mathbf{I}$, one can consider $\hat{\sigma}^2$ as an estimate of σ, where

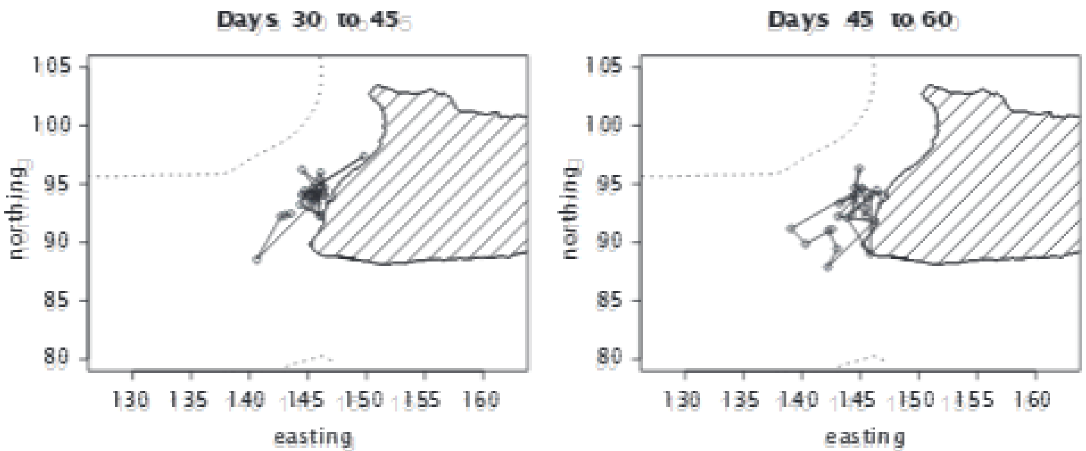
$$2\hat{\sigma}^2 \;=\; \frac{1}{I}\sum_i ||\mathbf{r}(t_{i+1} - \mathbf{r}(t_i) - \hat{\boldsymbol{\mu}}(\mathbf{r}(t_i))(t_{i+1} - t_i)||^2/(t_{i+1} - t_i),$$

$$d\mathbf{r}(t) \;=\; \boldsymbol{\mu}(\mathbf{r}(t))dt \;+\; \sigma d\mathbf{B}(t), \qquad \mathbf{r}(t) \;\in\; F$$

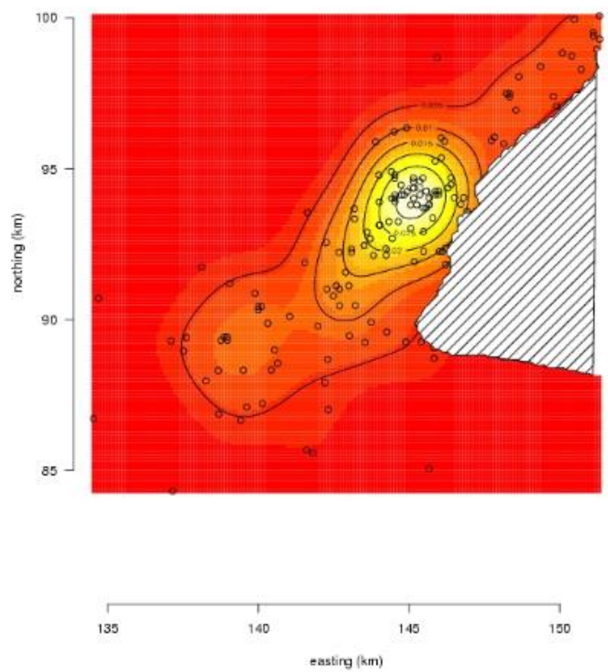
will be employed, with $\mathbf{B}$ Brownian, and with the motion restricted to within F, the 200 fathom region.



Second seal male



$$H(x, y) = \beta_{10}x + \beta_{01}y + \beta_{20}x^2 + \beta_{11}xy + \beta_{02}y^2 + C/d_M(x, y)$$



Discussion.

Particular marine habitats attractors to foraging monk seals. Here foraging habitats confined to relatively shallow offshore bathymetric features
(i.e. less than 200 fathoms deep - Penguin Bank)

Time seal spent foraging appeared constrained by powerful attractor associated with periodic resting ashore (i.e., terrestrial haulout habitat).

Example 3. Elk (*cervus elaphus*)

US Federal land managers examined effects on Rocky Mountain elk of forest management, domestic livestock grazing

Starkey Project initiated in northeastern Oregon
    9000 ha fenced area

Experiments using locations of elk, deer and cattle
    continuously monitored

Problem of interest: description of movement of free-ranging animals.

ModelI. gradient system (Skorokhod).

$$d\mathbf{r}_i = -\sum_{j \neq i} \nabla V(\mathbf{r}_i - \mathbf{r}_j)\, dt + \sigma\, d\mathbf{B}_i .$$

Note $\mathbf{r}_i - \mathbf{r}_j$

The potential function form employed is

$$\beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 xy + \beta_5 y^2 + \beta_6 x^3 + \beta_7 x^2 y + \beta_8 xy^2 + \beta_9 y^3$$

with (x,y) denoting location. The gradient is

$$\begin{pmatrix} 1 & 0 & 2x & y & 0 & 3x^2 & 2xy & y^2 & 0 \\ 0 & 1 & 0 & x & 2y & 0 & x^2 & 2xy & 3y^2 \end{pmatrix}$$

matrix multiplied by the transpose of the row vector

$$(\beta_1\ \beta_2\ \beta_3\ \beta_4\ \beta_5\ \beta_6\ \beta_7\ \beta_8\ \beta_9)$$

$\mathbf{X}\boldsymbol{\beta}^{\mathbf{T}}$: linear combination of β's

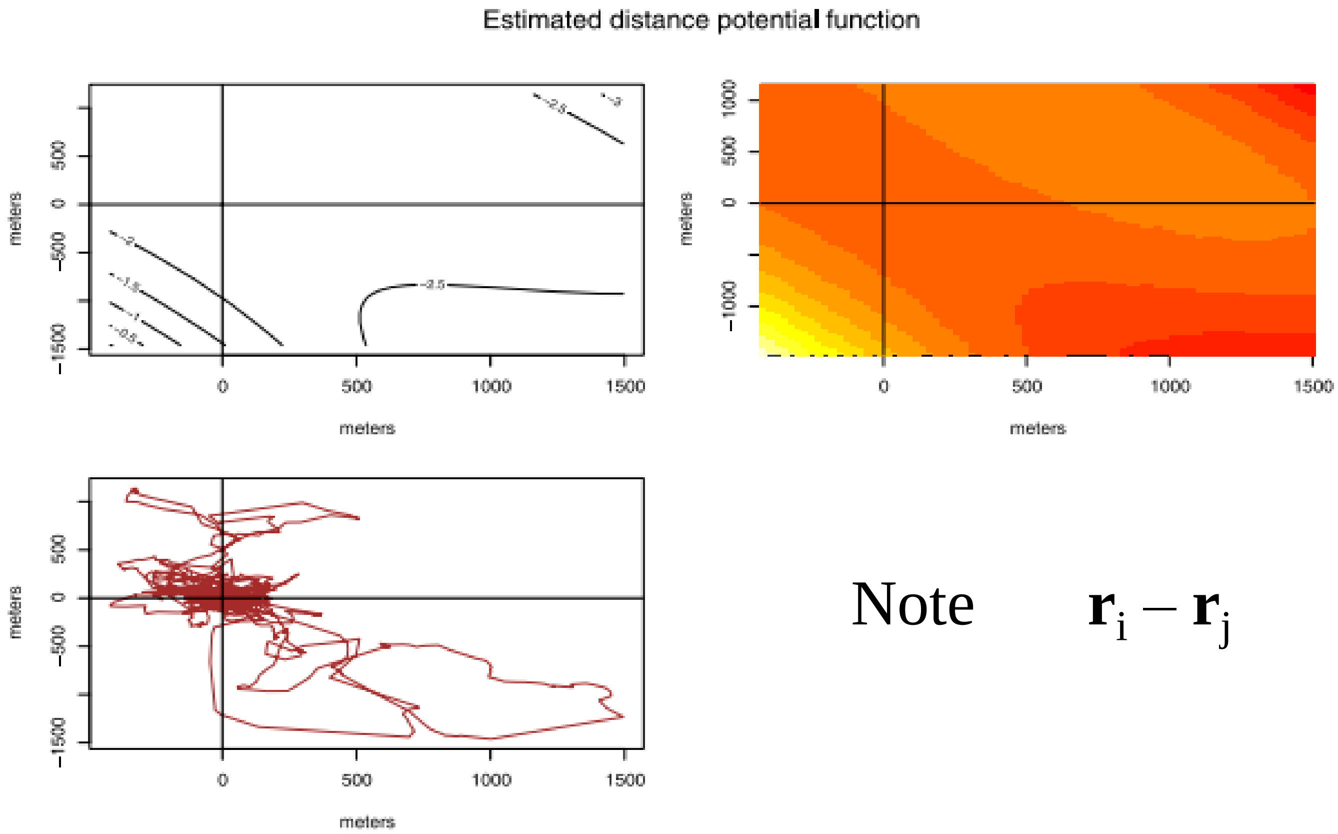


Figure 6 *Elk 398's movement with respect to elk 395.*

Note $\mathbf{r}_i - \mathbf{r}_j$

Attraction plus bias to SE

ModelIII. gradient syetem

The model is now

$$d\mathbf{r}_i = -\sum_{j\neq i} \frac{\nabla W(\|\mathbf{r}_i - \mathbf{r}_j\|)}{\|\mathbf{r}_i - \mathbf{r}_j\|} dt + \sigma\, d\mathbf{B}_{ij}$$

for some real-valued function W of real values.

Note $\|\mathbf{r}_i - \mathbf{r}_j\|$

Distance apart

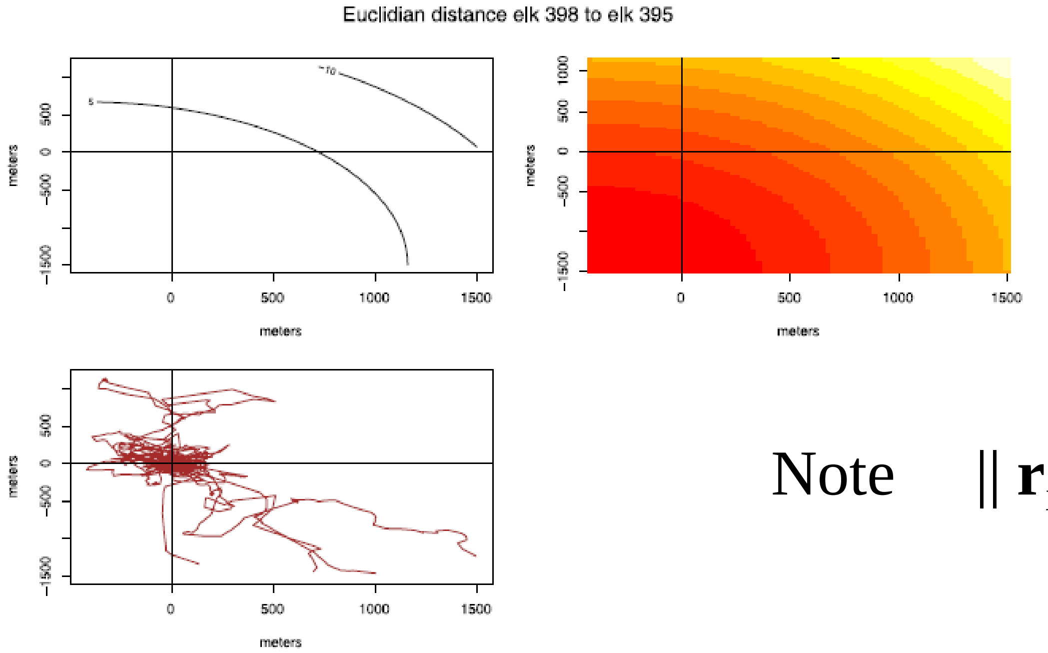


Figure 7 *The results of fitting Model III with elk 398 dependent and 395 explanatory.*

Note $\| \mathbf{r}_i - \mathbf{r}_j \|$

Expanding layers

Discusion

The last two models concern a particle being influenced by another particle of the same type or by a lagged particle of a different type.

Further models.

$$d\mathbf{r}_i(t) = -\nabla U_i(\mathbf{r}_i(t))\,dt - \sum_{j \neq i} \nabla V_{ij}\big(\mathbf{r}_i(t) - \mathbf{r}_j(t)\big)\,dt + \sigma\,d\mathbf{B}_i(t)$$

$$d\mathbf{r}(t) = \mu(\mathbf{r}(t))dt + \nu(|\mathbf{r}(t) - \mathbf{x}(t-\tau)|)dt + \sigma d\mathbf{B}(t).$$

Two models concerning particle being influenced by another of same type or by a lagged particle of different type (hunter).

Example 4 driftingtag.pdf

Based on surface drifting movement of a small satellite-linked radio transmitter tag.

Goal: to compare its movements (direction and velocity with direction and velocity of sea surface currents estimated independently from gradient of sea surface height..

Daily estimates of th tag's locations determined from transmissions received at irregular times by polar-orbittinf satellites

Second goal developing presictive model using past tag locations, the currents and winds

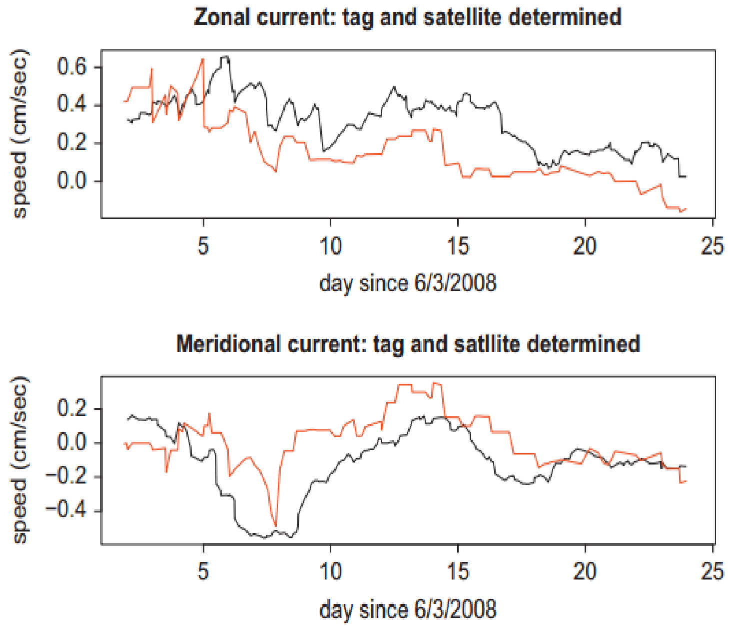


Fig. 3. The black lines provide the biweighted tag values (10). The top panel provides the east–west, i.e. zonal current and the bottom for the north–south, i.e. meridional. The red lines provides the geostrophic current values. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Surface winds may be introduced into the model by including a term X_W

$$(r(t_{i+1}) - r(t_i))/(t_{i+1} - t_i) = \alpha + \beta_C X_C(r(t_i), t_i) + \beta_W X_W(r(t_i), t_i) + \sigma Z_{i+1}/\sqrt{(t_{i+1} - t_i)}$$

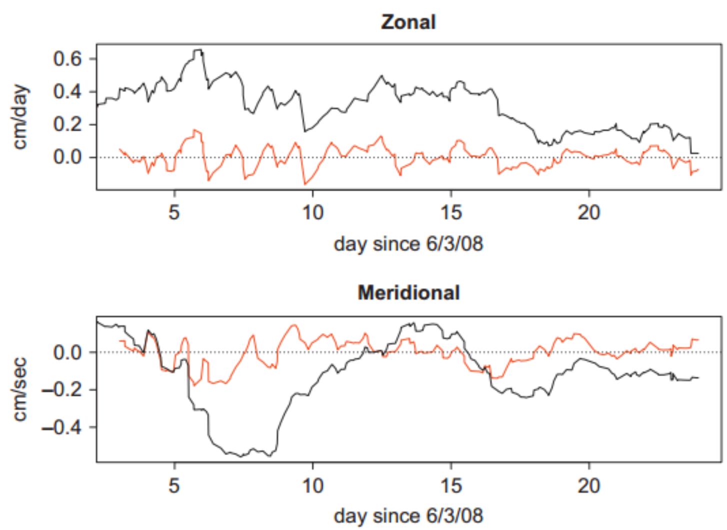
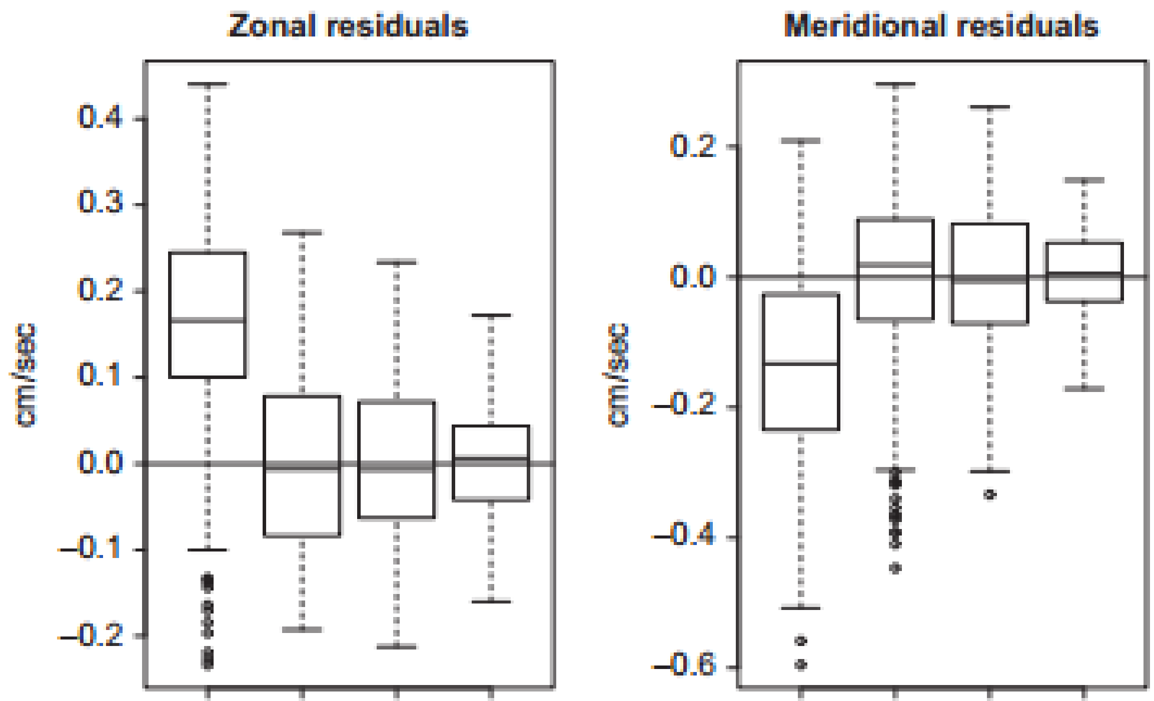


Fig. 5. Results of fitting model (12) (i.e. including current, wind, and average of locations for the tag during the preceding 24 h).

form is $V(r) = \gamma_1 x + \gamma_2 y + \gamma_{11} x^2 + \gamma_{12} xy + \gamma_{22} y^2 + C/dM$

where $dM = dM(x, y)$ is the distance from location (x, y) to the nearest point of a region, M,



Uses

intervention analysis
prediction
change
association
residual assessment
regression results
. explanatories
measurement error
…

Difficulties.

Boundaries and other objects

Island/line – use nearest point

slopes

Outliers

Different (iregular) times for different animals

Future work variance stabilizing transform

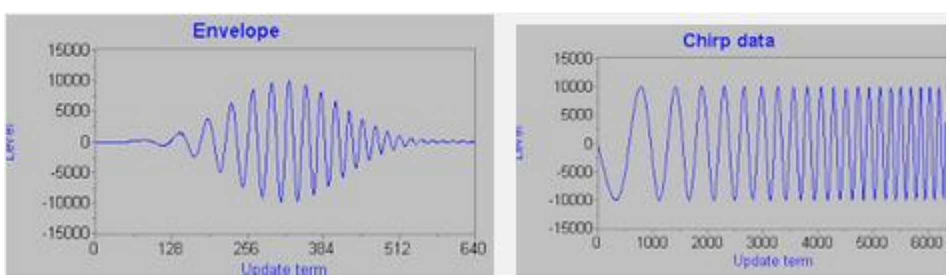
1. Acceleration model (Data anyone?)

2. Effects of sound – add terms to movement model

   e.g. moving (pressure) wave $g(\alpha x + \beta y - \gamma t)$

   Does g have an effect? Is there change?

   Example filtered sonar signal



3.Study many animals at the same time (herd)

Summary.

EDA discoveries: great circle, Penguin Bank, clustering, testing NOAA values, Brownian with trend on sphere

elephant seal
SDE model

monk seal
potential function model

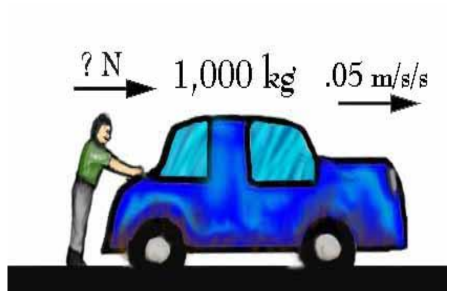
two elk
three potential models

free floating tag
explanatories derived from a potential

Acknowledgments

I thank my collaborators A. Ager, C. Littnan, H. Preisler, and B. Stewart

Newton’s Second Law [PUT TO END?]



Langevin (1908) “… trajectory of a particle …”

$$\mathbf{v} = d\mathbf{R}/dt, \quad m d\mathbf{v}/dt = -\,\gamma\,\mathbf{v} + \boldsymbol{\Psi}(t)$$

particle:	marine mammal,	m: mass	
γ:	friction coefficient:	$d\mathbf{v}/dt$:	acceleration
$m\, d\mathbf{v}/dt$:	momentum:	$\boldsymbol{\Psi}$:	random forces

Chandresekhar (1943) adds potential

$$\mathbf{K}(\mathbf{R}(t),t) = -\,(\delta/\delta x,\ \delta/\delta y)\text{Б}(\mathbf{R(i)},t)$$

Stochastic calculus

t: continuous time t_i : increasing discrete times

$\mathbf{r}(t)$: location at time t

Random walk

$$\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i) = IN_2 (0, (t_{i+1} – t_i)\boldsymbol{\Sigma})$$

Brownian $\mathbf{B}(t)$ t continuous

disjoint increments $\mathbf{B}(I) \sim IN(0, |I|)$

$$d\mathbf{Y}(t) = \mathbf{Y}(t+dt) – \mathbf{Y}(t)$$

Random walk with linear drift SDE

$$d\mathbf{Y}(t) = (\boldsymbol{\alpha} + \boldsymbol{\beta}t)dt + \boldsymbol{\Sigma}\, d\mathbf{B}(t)$$

Variance-stabilizing transform

diagonal and depends on $\mathbf{r}$. Specifically Ait-Sahalia (2008) presents conditions under which a process described by

$$d\mathbf{r} = \mu(\mathbf{r})\,dt + \Sigma(\mathbf{r})\,d\mathbf{B}$$

can be transformed into one satisfying

$$d\mathbf{r} = \mu(\mathbf{r})\,dt + d\mathbf{B}$$

by an invertible infinitely differentiable function ν satisfying $\nabla\nu(\mathbf{r}) = \Sigma^{-1}(\mathbf{r})$.

Details of computations

Robust/resistant methods

EDA – visualization

lm, mgcv

Appendix

Estimates large sample properties of estimates follow from existing results (Appendix)

$r(t_{k+1}) - E\{r(t_{k+1}) | F_k\}$, k= 0, 1, 2,...

martingale difference series F_k pertinent sequence of σ -fields

$$\mu(\mathbf{r}) = g(\mathbf{r})^T \beta$$

r β and a p by L known function g. This assu
linear regression model

$$\mathbf{Y}_n = \mathbf{X}_n \beta + \epsilon_n$$

1 values $(\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i))/\sqrt{t_{i+1} - t_i}$ to form the
ces $\mu(\mathbf{r}(t_i), t_i)\sqrt{(t_{i+1} - t_i)}$ to form the $(n-1)p$
$\sigma \mathbf{Z}_{i+1}$ to form ϵ_n. One is thereby led to consid

$$\hat{\beta} = (\mathbf{X}_n^T \mathbf{X}_n)^{-1} \mathbf{X}_n^T \mathbf{Y}_n$$

inverse exists. Continuing, one is led to estima
jth entry of $\mathbf{Y}_n$ and $\mathbf{x}_j^T$ denote the jth row of $\mathbf{X}$

$$= ((n-1)p^{-1} \sum (y_j - \mathbf{x}_j^T \hat{\beta})^T (y_j - \mathbf{x}_j^T \hat{\beta}),$$

desired, proceed to form approximate confide
esults of [25]. In particular, the distribution of

$$(g(\mathbf{r})^T (\mathbf{X}_n^T \mathbf{X}_n)^{-1} g(\mathbf{r}))^{-1/2} g(\mathbf{r})^T (\hat{\beta} - \beta)/s_n$$

$$\hat{\delta} = -0.0113 \text{ rad/day} = -72.0 \text{ km/day},$$

$$\hat{\sigma} = 0.00805 \text{ rad/day} = 51.3 \text{ km/day}.$$

d error of $\hat{\delta}$ is 0.0011.

$$\theta_t' = \theta_t + \tau\varepsilon_t'$$

$$\phi_t' = \phi_t + \tau\gamma_t'/\sin\theta_t'$$

$$\theta_{t+1} - \theta_t = \frac{\sigma^2}{2\tan\theta_t} - \delta + \sigma\varepsilon_{t+1}$$

$$\phi_{t+1} - \phi_t = \frac{\sigma}{\sin\theta_t}\gamma_{t+1}$$

$$\hat{\delta} = .0126(.0001)$$

$$\hat{\delta}_1 = .0109(.0001)$$

$$\hat{\sigma} = .000489(.000004)$$

Residuals

$$\frac{(\sin\tilde{\theta}_t)(\tilde{\phi}_{t+1} - \tilde{\phi}_t)}{\hat{\sigma}}$$

ınit variance independent Gaussian noi

ses correspond to measurement error.

ıdel (6.3-4) for the case of no measurer

ıood. The values obtained are:

$$\hat{\delta} = .0112(.0011)\textit{radians}$$

$$\hat{\sigma} = .00805\textit{radians}$$

Measurement error

$$\theta_t' = \theta_t + \epsilon_t',$$

$$\phi_t' = \phi_t + \eta_t'/\sin\theta_t'$$

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$$\hat{\tau} = .0175(.0011)$$

δ speed towards the origin.

Potential function

$$H(\varphi,\theta) = \tfrac{1}{2}\sigma^2 \log\sin\theta - \delta\theta$$

point of attraction - North Pole